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# *COHERENCE QED IN COLD FUSION*

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# Introduction

*In the usual QED of Condensed Matter the start point is:*

$$H_{matt} = \sum_{j=1}^{N} H_0(\vec{x}_j, \vec{p}_j, \alpha_j) + V(\vec{x}_1, \alpha_1, \ldots, \vec{x}_N, \alpha_N)$$

*where (generally) V is a short range potential.*

*Introducing the orthonormal complete set of vector that diagonalizes $H_0$,*

*we have*

$$\{\phi_n(\vec{x}, \alpha)\}$$

Fock-space

$$H_0 = \sum_{j=1}^{N} H_0(\vec{x}_j, \vec{p}_j, \alpha_j) = \sum_n \varepsilon_n a_n^+ a_n$$

$$E\{n_m\} = \sum_m n_m \varepsilon_m$$

$$\Psi(\vec{x}, \alpha, t) = \sum_n a_n(t) \phi_n(\vec{x}, \alpha)$$

## QED Lagrangian

*By means of field operator we have*

$$L_{matt} = i\int_{\vec{x},\alpha} \Psi^{+}(\vec{x},\alpha,t)\Psi_{t}(\vec{x},\alpha,t) - H_0 \qquad [\Psi(\vec{x},\alpha,t),\Psi^{+}(\vec{x}',\alpha',t)]_{\pm} = \delta(\vec{x}-\vec{x}')\delta_{\alpha\alpha'}$$

$$N = \int_{\vec{x},\alpha} \Psi^{+}(\vec{x},\alpha,t)\Psi(\vec{x},\alpha,t) = \sum_{n} a_n^{+} a_n$$

*Now we want introduce in the QED the essential role played by e.m. interactions in determining the structure and the properties of the innumerable systems of Condensed Matter*

$$\vec{p}_i \rightarrow \vec{p}_i + e_i \vec{A}(\vec{x}_i, t)$$

$$H_{matt} = \sum_{j=1}^{N} H_0(\vec{x}_j, \vec{p}_j, \alpha_j) + H^1_{rad} + H^2_{rad} + V(\vec{x}_1, \alpha_1, \ldots, \vec{x}_N, \alpha_N)$$

# Coherence QED

$$H_{matt} = \sum_{j=1}^{N} H_0(\vec{x}_j, \vec{p}_j, \alpha_j) + H^1_{rad} + H^2_{rad} + H_{SR}$$

$$H^1_{rad} = e \int_{\vec{x},\alpha} \vec{A}(\vec{x},t)\Psi^+(\vec{x},\alpha,t)\vec{J}(\alpha)\Psi(\vec{x},\alpha,t)$$

$$H^2_{rad} = e^2\lambda \int_{\vec{x},\alpha} \vec{A}^2(\vec{x},t)\Psi^+(\vec{x},\alpha,t)\Psi(\vec{x},\alpha,t)$$

$$H_{SR} = V(\vec{x}_1,\alpha_1,\dots,\vec{x}_N,\alpha_N)$$

***These two radiative terms make the standard QED a new QED***

***Z-electron example for understand the role of $\lambda$***

$$H = \sum_{r=1,Z} \frac{(p_r - eA(x_r,t))^2}{2m_e} \Rightarrow H_0 - \frac{e}{m_e}\sum_{r=1,Z} p_r A(x_r,t) + \frac{e^2}{2m_e}\sum_{r=1,Z} A^2(x_r,t)$$

$$J = -\sum_{r=1,Z} \frac{p_r}{m_e}$$

$$\lambda = \frac{Z}{2m_e}$$

$$H = H_0 + eJA(x_r,t) + e^2\lambda A^2(x_r,t)$$

# Coherence QED

## Lagrangian of E.M. Field

$$\vec{A}(\vec{x},t)=\sum_{\vec{k}r}\frac{1}{\sqrt{2\omega_{\vec{k}}V}}\left[a_{\vec{k}r}(t)\vec{\varepsilon}_{\vec{k}r}e^{-i\omega_k t}e^{i\vec{k}\cdot\vec{x}}+a_{\vec{k}r}{}^{*}(t)\vec{\varepsilon}^{\,*}{}_{\vec{k}r}e^{i\omega_k t}e^{-i\vec{k}\cdot\vec{x}}\right]$$

$$L_{em}=\sum_{\vec{k}r}\left[\frac{i}{2}\left(a_{\vec{k}r}(t)^{*}\dot{a}_{\vec{k}r}(t)-\dot{a}_{\vec{k}r}(t)^{*}a_{\vec{k}r}(t)\right)+\frac{1}{2\omega_{\vec{k}}}\dot{a}_{\vec{k}r}(t)^{*}\dot{a}_{\vec{k}r}(t)\right]$$

$$L_{matt}=i\int_{\vec{x},\alpha}\Psi^{+}(\vec{x},\alpha,t)\Psi_{t}(\vec{x},\alpha,t)-H_{matt}$$

$$L_{QED-Coherence}=L_{matt}+L_{e.m.}$$

***The existence of a large number N suggest rescaling theory so as to make N appear explicitly in the path-integral. We thus define:***

$$\Psi_0(\vec{x},\alpha,t)=\frac{1}{\sqrt{N}}\Psi\,(\vec{x},\alpha,t) \qquad a^0_{\vec{k}r}=\frac{1}{\sqrt{N}}a_{\vec{k}r} \qquad \int_{\vec{x},\alpha}\Psi_0^{\,+}(\vec{x},\alpha,t)\Psi_0(\vec{x},\alpha,t)=1$$

**After rescaling of the lagrangians we can derive the motion equation (by path integral approach) by means of the principle of extremal action**

$$\begin{cases}\Psi_0(\vec{x},\alpha,t)=\varphi(\vec{x},\alpha,t)+\dfrac{1}{\sqrt{N}}\eta(\vec{x},\alpha,t)\\ a^0_{\vec{k}r}=\alpha_{\vec{k}r}+\dfrac{1}{\sqrt{N}}\beta_{\vec{k}r}(t)\end{cases} \Rightarrow \quad \delta\int_{t_i}^{t_f}(L_{matt}+L_{em})=0$$

**Coherence equation**

$$\begin{cases} i\dfrac{\partial\varphi}{\partial t}=H_0\varphi+e\sqrt{N}\vec{A}_0\cdot\vec{J}\varphi \\ -\dfrac{1}{2\omega_{\vec{k}}}\ddot{\alpha}_{\vec{k}r}+i\dot{\alpha}_{\vec{k}r}-\dfrac{e^2}{\omega_{\vec{k}}}\left(\dfrac{N}{V}\right)\lambda\alpha_{\vec{k}r}=e\dfrac{1}{\sqrt{2\omega_{\vec{k}}}}\left(\dfrac{N}{V}\right)^{1/2}\vec{\varepsilon}^{\,*}_{\vec{k}r}e^{i\omega_{\vec{k}}t}\displaystyle\int_{\vec{x}\alpha}e^{-i\vec{k}\vec{x}}\varphi^*\vec{J}\varphi \end{cases}$$

## *QED-Coherence of TWO-LEVEL SYSTEMS*

**In this case we have:**

$$\{\varphi_n(\vec{x},\alpha)\} = \{\varphi_1(\vec{x},\alpha), \varphi_2(\vec{x},\alpha)\} \qquad \begin{cases} H\varphi_1 = E_1\varphi_1 \\ H\varphi_2 = E_2\varphi_2 \end{cases} \qquad E_1 - E_2 = \omega_0$$

**Put** $J_i = \delta_{i1} J \sigma_1$ **and using the interaction representation** $\varphi_i(\vec{x},t) = e^{-iE_i t}\chi_i(\vec{x},t)$

**we obtain the following *coherence equation***

$$i\frac{\partial \chi_2(\vec{x},t)}{\partial t} = -\frac{\vec{\nabla}^2}{2m}\chi_2(\vec{x},t) + eJ\sqrt{\frac{N}{V}}\frac{1}{\sqrt{2\omega_0}}\sum_{|\vec{k}|=\omega_0, r}\vec{\varepsilon}^{\,*}_{\vec{k}r}\alpha^*_{\vec{k}r}e^{-i\vec{k}\vec{x}}\chi_1(\vec{x},t)$$

$$i\frac{\partial \chi_1(\vec{x},t)}{\partial t} = -\frac{\vec{\nabla}^2}{2m}\chi_1(\vec{x},t) + eJ\sqrt{\frac{N}{V}}\frac{1}{\sqrt{2\omega_0}}\sum_{|\vec{k}|=\omega_0, r}\vec{\varepsilon}^{\,*}_{\vec{k}r}\alpha^*_{\vec{k}r}e^{-i\vec{k}\vec{x}}\chi_2(\vec{x},t)$$

$$-\frac{1}{2\omega_0}\ddot{\alpha}_{\vec{k}r} + i\dot{\alpha}_{\vec{k}r} - \frac{e^2}{\omega_0}\left(\frac{N}{V}\right)\lambda\alpha_{\vec{k}r} = eJ\frac{1}{\sqrt{2\omega_0}}\left(\frac{N}{V}\right)^{1/2}\vec{\varepsilon}^{\,*}_{\vec{k}r}\int_{\vec{x}} e^{-i\vec{k}\vec{x}}\chi_2^*\chi_1$$

$$\alpha_{\vec{k}r} = 0 \qquad \text{for} \qquad |\vec{k}| \neq \omega_0$$

## *Coherence domains*

***In the last equation, we can observe that the resonating modes whose vacuum frequencies $\omega_k = \omega_0$ are strongly coupled with the matter field $\chi$. It produce the remarkable result that the e.m. and matter field exhibit a space-structure comprising an array of COHERENCE DOMAINS (CD)***

***CD size=$2\pi/\omega_0$***

**Now if we work within a CD, we can write** $\chi_i(\vec{x},t) \approx \frac{1}{\sqrt{V_{CD}}}\chi_i(t)$ **so we have the following coherence equation for a CD:**

$$i\dot{\chi}_2(t) = eJ\sqrt{\frac{N}{V}}\frac{1}{\sqrt{2\omega_0}}\sum_r \int d\Omega_{\vec{k}}\, \varepsilon^*_{\vec{k}r,1}\alpha^*_{\vec{k}r}\chi_1(t)$$

$$i\dot{\chi}_1(t) = eJ\sqrt{\frac{N}{V}}\frac{1}{\sqrt{2\omega_0}}\sum_r \int d\Omega_{\vec{k}}\, \varepsilon_{\vec{k}r,1}\alpha_{\vec{k}r}\chi_2(t)$$

$$-\frac{1}{2\omega_0}\ddot{\alpha}_{\vec{k}r} + i\dot{\alpha}_{\vec{k}r} - \frac{e^2}{\omega_0}\left(\frac{N}{V}\right)\lambda\alpha_{\vec{k}r} = eJ\frac{1}{\sqrt{2\omega_0}}\left(\frac{N}{V}\right)^{1/2}\vec{\varepsilon}^{\,*}_{\vec{k}r,1}\chi^*_2(t)\chi(t)_1$$

$$|\chi_1|^2 + |\chi_2|^2 = 1$$

## Coherence domains

**Putting**

$$A = \sum_r \left(\frac{3}{8\pi}\right)^{1/2} \int d\Omega_{\vec{k}} \varepsilon_{\vec{k}r,1} \alpha_{\vec{k}r} \qquad g = eJ\left(\frac{8\pi}{3}\right)^{1/2}\left(\frac{N}{2V\omega_0^3}\right)^{1/2} \qquad \mu = \frac{e^2\lambda}{\omega_0^2}\left(\frac{N}{V}\right)$$

**We can write the coherence equation within CD in this way**

$$\dot{\chi}_2 = -igA^*\chi_1 \qquad \dot{\chi}_2 = -igA^*\chi_2 \qquad \frac{i}{2}\ddot{A} + \dot{A} + i\mu A = -ig\chi_2^*\chi_1$$

***If we derive the e.m. equation we will have***

$$\frac{i}{2}\dddot{A} + \ddot{A} + i\mu\dot{A} + gA = 0$$

$$\frac{p^3}{2} - p^2 - \mu p + g^2 = 0 \qquad g^2 > g_c^2 \qquad g_c^2 = \frac{8}{27} + \frac{2}{3}\mu + \left[\frac{4}{9} + \frac{2}{3}\mu^2\right]^{\frac{3}{2}}$$

***Finally we have these coserved quantities***

$$Q = A^*A + \frac{i}{2}(A^*\dot{A} - \dot{A}^*A) + \chi_1^*\chi_1^* \qquad H = Q + \frac{1}{2}\dot{A}^*\dot{A} + \mu A^*A + g\left(A^*\chi_2^*\chi_1 + A\chi_1^*\chi_2\right)$$

$$1 = \chi_1^*\chi_1 + \chi_2^*\chi_2$$

## Coherence QED of a plasma

$$H_0 = \sum_i \frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2} m\omega_p \xi_i^2$$

$$H^1_{rad} = e\int \Psi^+(\vec{x},\xi,t)\vec{J}\Psi(\vec{x},\xi,t)\vec{A}(\vec{x},t)$$

$J=Q(p/m)$

**If we resolve the CE for a plasma we obtain with μ=0**

$$g^2 = \left(\frac{2\pi}{3}\right) \Rightarrow g_c^2 = \frac{16}{27} < g^2 = \left(\frac{2\pi}{3}\right)$$

***But this solution don't satisfy the conserved quantities***

$$Q = \sum_k \left( A_k^* A_k + \frac{i}{2}(A_k^* \dot{A}_k - \dot{A}_k^* A_k) + \chi_k^* \chi_k^* \right)$$

$$H = \frac{E}{N\omega_p} = Q + \sum_k \left[ \frac{1}{2} \dot{A}_k^* \dot{A}_k - ig\left(A_k^* \chi_k - A_k \chi_k^*\right) \right]$$

$$\chi_k \prec e^{i\psi}$$

$$A_k \prec e^{i\phi}$$

$$\Rightarrow 1 - \dot{\phi} + \frac{g^2}{\dot{\phi}^2} = 0$$

***No Solution!***

## *Coherence QED of a plasma*

***We must introduce another interaction term***

$$H_0^R = \sum_i \frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2} m\omega_p \xi_i^2 + \omega_R \left( \sum_k \frac{a_k^+ a_k}{\alpha_{\max}^2} \right)$$

$$\alpha_{\max} = \sqrt{m\omega_p} \left( \frac{V}{N} \right)^{1/3}$$ and $\omega_R = 1.88\,\omega_p$

**The effect of this term is to strongly suppress coherent oscillation amplitudes when**

$$\alpha^2 \geq \alpha_{\max}^2$$

***In this way, we obtain as solution a e.m. field with a new frequency***

$$\omega_r = \omega_R (1 - \dot{\phi}) \quad \Rightarrow \quad \dot{\phi} = 1 + \sqrt{1 - 2g \frac{\chi}{A}}$$

## *Cold fusion phenomenon*

### *Why the $D_2$ on the PD surface became $D_2^+$?*

**Now we propose an explanation about the remarkable property of Pd to ionize $D_2$ or $H_2$ gas (need 30 eV!)**

**Due to the plasma in the lattice (10 electrons in the d-band) we have an e.m. field**

$$\nabla^2 \vec{A} = 0$$

**If we work in semi-space z>0**

$$\vec{A}(x, y, z) = \vec{A}(x, y, 0)e^{-\eta z} \qquad \eta = \sqrt{\omega_p^2 - \omega_r^2} > 0$$

**If we suppose that the system $D_2$-molecule can be reduced at a two-dimensional system ground and ionized we have (D is the dipole electric operator):**

$$|\chi\rangle = c_1|0\rangle + c_2|1\rangle$$ *and this interaction term* $$H_{\text{int}} = -\vec{E} \cdot \vec{D}$$

## Cold fusion phenomenon

**So put**

$$\vec{E} \approx \vec{u}\sin(\omega_r t)\sqrt{\frac{2N\omega_r^2}{V\omega_p}}\sqrt{\frac{2\pi}{3}}A \qquad \omega^* = \sqrt{\frac{2N\omega_r^2}{V\omega_p}}\sqrt{\frac{2\pi}{3}}Ae\left|\langle i|H_{\text{int}}|0\rangle\right| \qquad \left|\langle i|H_{\text{int}}|0\rangle\right| = 1\overset{0}{A}$$

$$i\frac{\partial}{\partial t}|\chi\rangle = \left[\frac{\omega_i}{2}\sigma_3 + \frac{\omega^*}{2}\left(e^{-i\omega_r t}\sigma_+ + e^{-i\omega_r t}\sigma_-\right)\right]|\chi\rangle$$

$$\omega^* \approx 10eV \qquad \omega_i \approx 30eV \qquad \omega_r \approx 0.2\omega_p \qquad \frac{N}{V} = 6.5\cdot 10^{22}\,cm^{-3} \qquad \omega_p \approx 10\,eV$$

$$P_i(t) = \frac{\omega^{*2}}{(\omega_i - \omega_r)^2 + \omega^{*2}}\sin^2\sqrt{(\omega_i - \omega_r)^2 + \omega^{*2}}\,\frac{t}{2}$$

*The average probability that a $D_2$-molecule be ionised on the surface of Pd is of the order of 5%!*

## *Tunneling Effect Probability within lattice*

$$|P|^2_{\text{int}} = \exp\left(-2\int_0^{\alpha} K(r)_{\text{int}}\, dr\right) \qquad K(r)_{\text{int}} = \sqrt{2\mu\left[E - V(r)\right]/\hbar^2}$$

where α is about 0.10 Å. In this case:

$$V(r)_{\text{int}} = k\frac{q^2}{r}\cdot\left[V(r)_M - J\frac{\xi\hbar\omega_r R}{r}\right]$$

→ ***is the contribute due to QED-Coherence plasma***

R *is the nuclear radius, ξ a parameter varying between 0.015 and 0.025 which depends on the structural characteristics of the lattice, T the absolute temperature at which the metal is placed experimentally.*

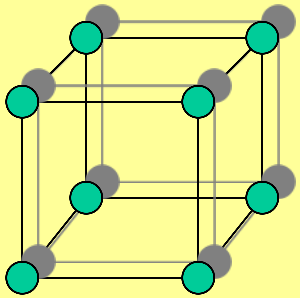
**In this case, the Morse potential will be:**

$$V(r)_M = \left( J / \varsigma \right) \exp\left( -2\varphi\left( r - r_0 \right) \right) - 2\exp\left( -\varphi\left( r - r_0 \right) \right)$$

**φ and ζ depend on the dynamic conditions of the system, while $r_0$ is the point of classic inversion.**

## Cold fusion phenomenon

**$J$ = impurities concentration**



$$\lambda_f = \lambda \frac{4\pi\rho\hbar}{\mu_d}\langle\frac{1}{p}\rangle$$ = **rate fusion for energy**

$$D_L = \iiint_\omega J \frac{\rho l^2 \nu \mathrm{b}^2}{\alpha 2hR} \exp\left(-\frac{U_0}{kT}\right) d\omega$$ = **3D lattice deformation**

$$\Gamma \approx \frac{\exp\left(-2\int_0^\alpha \mathrm{K(r)_{int}}\, dr\right)}{\lambda \cdot \frac{4\pi\rho\hbar}{\mu_d} \cdot \langle\frac{1}{p}\rangle} \cdot D_L$$

## Cold fusion phenomenon

# Tunneling Effect probability + 3D lattice interaction

**Palladium** $J \approx 0.75\%$ T(K) $\approx$ 100-300K $\alpha \approx .10$Å $\lambda = 10^{-3}$ eV / min $M_{Pd}$ /($\mu g$)

| T ≈ | 100 K | 140 K | 180 K | 220 K | 260 K | 300 K |
|---|---|---|---|---|---|---|
| E (eV) | Γ ≈ | Γ ≈ | Γ ≈ | Γ ≈ | Γ ≈ | Γ ≈ |
| 150 | $10^{-77}$ | $10^{-67}$ | $10^{-71}$ | $10^{-63}$ | $10^{-60}$ | $10^{-59}$ |
| 160 | $10^{-73}$ | $10^{-65}$ | $10^{-70}$ | $10^{-61}$ | $10^{-56}$ | $10^{-55}$ |
| 170 | $10^{-71}$ | $10^{-62}$ | $10^{-69}$ | $10^{-58}$ | $10^{-55}$ | $10^{-52}$ |
| 180 | $10^{-69}$ | $10^{-60}$ | $10^{-67}$ | $10^{-54}$ | $10^{-53}$ | $10^{-50}$ |
| 190 | $10^{-68}$ | $10^{-54}$ | $10^{-64}$ | $10^{-53}$ | $10^{-52}$ | $10^{-45}$ |
| 200 | $10^{-64}$ | $10^{-53}$ | $10^{-63}$ | $10^{-51}$ | $10^{-50}$ | $10^{-43}$ |
| 210 | $10^{-63}$ | $10^{-54}$ | $10^{-64}$ | $10^{-50}$ | $10^{-48}$ | $10^{-40}$ |
| 220 | $10^{-61}$ | $10^{-52}$ | $10^{-63}$ | $10^{-49}$ | $10^{-45}$ | $10^{-35}$ |
| 230 | $10^{-60}$ | $10^{-50}$ | $10^{-62}$ | $10^{-47}$ | $10^{-43}$ | $10^{-29}$ |
| 240 | $10^{-59}$ | $10^{-45}$ | $10^{-60}$ | $10^{-44}$ | $10^{-39}$ | $10^{-25}$ |
| 250 | $10^{-58}$ | $10^{-41}$ | $10^{-59}$ | $10^{-40}$ | $10^{-36}$ | $10^{-21}$ |

Morse potential

## Conclusion

→ Introducing in the QED the essential role played by e.m. interactions in the Condensed Matter we can predict new phenomenon

→ The Coherence Domains are the new structures of Condensed Matter

→ By means of this theory we can explain the ionisation of $H_2$ or $D_2$ molecule on the Pd surface

→ Integrating this theory with the microcrack formation we are able to explain the cold fusion phenomenon