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# Nominal and Parametric Self-Induction of Reactive Elements and Long Lines

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### Abstract

In the article are examined the laws of the self-induction of reactive elements. It is shown that such reactive elements as capacity and inductance with their connection to the current generators and voltage present the effective resistance, whose value depends on time. Is introduced the concept of the parametric self-induction of reactive elements and long lines, which occurs when their parameters they change in the time. Introduction of this concept made possible to calculate the velocity of propagation of constant stresses and currents in the long lines, what cannot be made with the use of wave equations. Is examined the behavior of the input resistance of the long line, whose parameters change with mechanical method. It is shown that this resistance depends on the rate of lengthening line and can acquire negative importance in the case of the decrease of the length of line.

## 1. Introduction

To the radio-technical elements with the lumped parameters relate effective resistance, capacity and inductance. Long lines relate to the elements with the distributed parameters. The parameters of the long lines are characterized by linear capacity and linear inductance, which determine line characteristic. By induction we will understand the reaction of the elements to the connection to them of the sources of voltage or current (further the power sources) indicated. The internal resistance of ideal the voltage sources strives zero, while the internal resistance of ideal current source approaches infinity. By parametric self-induction by induction we will understand the reaction of the indicated elements with the depending on the time parameters to the connection to them of the power sources.

Many specialists consider that the capacity and inductance are reactive elements and cannot take away energy in the alternating current circuits or voltage [1-4]. In order to be convinced of the fact that this is not always carried out, it suffices to conduct simple experiment. If we connect capacitor to the electric brush of alternating current, to and then open it, then it is possible to reveal that the capacitor is charged. If we repeat this experiment repeatedly, then it is possible to establish that a voltage drop across the terminals of capacitor varies in the limits from the zero values, to the amplitude value of a potential difference in the network. This fact attests to the fact that the capacitor can take away energy from the electric brush, since the energy, accumulated in the capacitor, is determined by the relationship

$$W_c = \frac{1}{2} CU^2$$

where  $U$  is voltage drop across the terminals of capacitor.

To load capacitor is possible and from the source of direct current, and in this case the energy, accumulated in the capacitor, will be obtained from this source. Idea about the fact that the reactive elements cannot take away energy in the alternating current circuits, was formed for that reason, that with the examination of the properties of reactive elements in such chains are examined the periodic processes of infinite duration and the moment of turning off of reactive element from the chain is not considered.

If we charge capacitor from the source of direct current, then a voltage drop across it will be grow, and it will derive energy from the source, accumulating it similar to storage battery. In this case the capacity can be represented as the effective resistance, which depends on the time. The same relates also to the inductance, connected to the dc power supply. But capacity and inductance can return energy into the external circuit, if their value changes.

The special features of capacity and inductance indicated make it possible to solve a question about the velocity of propagation of constant stresses and currents in the long lines, what cannot be made, using telegraphic of equations.

## 2. Reactive Self-Induction of the Capacity

If the capacity  $C$  is charged to a potential difference  $U$ , then the charge  $Q$ , accumulated in it, is determined by the relationship

$$Q_{c,U} = CU .$$

Magnitude of the charge can change with the method of changing the potential difference with a constant capacity, or with a change in the capacity with a constant potential difference.

The strength of current, which flows through the capacity, is determined by the relationship

$$I = \frac{dQ_{c,U}}{dt} = C \frac{\partial U}{\partial t} + U \frac{\partial C}{\partial t} .$$

Consequently, current in circuit can be obtained by two methods: changing stress on the capacity with a constant capacity or changing capacity with constant stress on it.

For the case, when the capacity  $C_0$  is constant, we obtain

$$I = C_0 \frac{\partial U}{\partial t} . \quad (2.1)$$

Let us connect to the capacity direct-current generator and we will support in it the direct current  $I_0$ . Then, after integrating relationship (3.1) with respect to the time, we obtain the dependence of stress on the capacity from the time.

$$U = \frac{I_0 t}{C_0} . \quad (2.2)$$

This relationship, which connects the direct current, which flows through the capacity, and stress on it is the analog of Ohm's law. For this reason the value

$$R = \frac{t}{C_0}$$

plays the role of effective resistance, which depends on time. The it should be noted that obtained result is completely obvious, however, such properties of capacity, which customary to assume by reactive element they were for the first time noted in the work [3].

The power, output by current source, is determined in this case by the relationship:

$$P(t) = \frac{I_0^2 t}{C_0} \quad (2.3)$$

the energy, accumulated by capacity in the time of , we will obtain, after integrating relationship (2.3) with respect to the time:

$$W_c = \frac{I_0^2 t^2}{2C_0} .$$

Substituting here the value of current from relationship (2.2), we obtain the dependence of the value of the accumulated in the capacity energy from the instantaneous value of stress on it:

$$W_c = \frac{1}{2} C_0 U^2 .$$

If we support at the capacity constant stress  $U_0$ , and to change capacity, then the current, which flows through it, is written down

$$I = U_0 \frac{\partial C(t)}{\partial t} \quad (2.4)$$

This case relates to the parametric self-induction.

This relationship, which connects the direct current, which flows through the capacity, and stress on it is the analog of Ohm's law. For this reason the value

$$R_c = \left( \frac{\partial C(t)}{\partial t} \right)^{-1}$$

plays the role of the effective resistance.

The power, expended in this case by source, is determined by the relationship:

$$P = U_0^2 \frac{\partial C(t)}{\partial t} .$$

From this relationship is evident that depending on the sign of derivative the expendable power can have different signs. When the derived positive, expended by source power proceeds with an increase in the energy, stored up in the

capacity. If derived negative, then work carries out the external source, which ensures the decrease of capacity. The energy, spent by this source, is separated in the external circuit.

### 3. Reactive Self-Induction of the Inductance

Let us examine the processes, proceeding in the inductance. Let us introduce the concept of the flow

$$\Phi_{L,I} = LI.$$

Stress on the inductance is equal to the derivative of the flow of current induction on the time:

$$U = \frac{d\Phi_{L,I}}{dt} = L \frac{\partial I}{\partial t} + I \frac{\partial L}{\partial t}.$$

Let us examine the case, when the inductance of is constant.

$$U = L_0 \frac{\partial I}{\partial t}. \quad (3.1)$$

Will maintain a constant voltage  $U_0$  across the inductance. Then, after integrating relationship (3.1) with respect to the time, we obtain

$$I = \frac{U_0 t}{L_0}. \quad (3.2)$$

This relationship, which connects the direct current, which flows through the capacity, and stress on it is the analog of Ohm's law. Therefore the value

$$R = \frac{L_0}{t}$$

plays the role of effective resistance, which depends on time.

The power, expended in this case by source, is determined by the relationship:

$$P(t) = \frac{U_0^2 t}{L_0}. \quad (3.3)$$

This power linearly depends on time. After integrating relationship (3.3) on the time, we will obtain the energy, accumulated in the inductance

$$W_L = \frac{1}{2} \frac{U^2 t^2}{L}.$$

After substituting in this relationship the value of stress from relationship (3.2), we obtain the energy, stored up in the inductance,

$$W_L = \frac{1}{2} L_0 I^2.$$

If we support in the inductance the direct current  $I_0$ , and to change inductance, then the current, which flows through it, will be written down

$$U = I_0 \frac{\partial L(t)}{\partial t}. \quad (3.4)$$

This case relates to the parametric self-induction. This relationship, which connects the direct current, which flows through the capacity, and stress on it is the analog of Ohm's law. Therefore the value

$$R = \frac{dL(t)}{dt}$$

plays the role of the effective resistance.

The power, expended in this case by source, is determined by the relationship:

$$P = I_0^2 \frac{\partial L(t)}{\partial t},$$

As in the case with the capacity, effective resistance can be both the positive and negative. This means that the inductance can how derive energy from without, so also return it into the external circuits.

### 4. Propagations of Constant Voltage and Current in the Long Lines

The processes of the propagation of voltages and currents in the long lines determine the wave equations [5-12]

$$\frac{\partial^2 U}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 U}{\partial t^2}$$

$$\frac{\partial^2 I}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 I}{\partial t^2},$$

which are obtained from the telegraphic equations

$$\frac{\partial U}{\partial z} = -L_0 \frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial z} = -C_0 \frac{\partial U}{\partial t}$$

where  $L_0$  and  $C_0$  linear inductance and the capacity of line.

The velocity of propagation in this line is determined by the relationship

$$v = \frac{1}{\sqrt{L_0 C_0}}$$

The knowledge second derivative voltages and currents is required during the solution of wave equations.

But wave equations do not give the answer to the question, as one should enter when to incoming line it is connected dc power supply. The results, obtained in two previous divisions, give answer to these questions. The processes,

examined in two previous paragraphs, concern chains with the lumped parameters, when the distribution of potential differences and currents in the elements examined can be considered uniform. However, there are chains, for example the long lines, in which the spatial distribution of voltages and currents they are not uniform.

Let us examine processes in the long line, whose capacity and inductance are the distributed parameters. We will consider that the linear capacity and the inductance of line is equal  $C_0$  and  $L_0$  respectively. If we to this line connect the dc power supply  $U_0$ , then its front will be extended in the line some by the speed  $v$ , and the moving coordinate of this front will be determined by the relationship  $z = vt$ . In this case the total quantity of the charged capacity and the value of the summary inductance, along which it flows current, calculated from the beginning lines to the location of the front of stress, will change according to the law:

$$C(t) = zC_0 = vt C_0,$$

$$L(t) = zL_0 = vt L_0.$$

The source of voltage  $U_0$  will in this case charge the being increased capacity of line, for which from the source to the charged line in accordance with relationship (2.4) must leak the current:

$$I = U_0 \frac{\partial C(t)}{\partial t} = vU_0 C_0.$$

This current there will be the leak through the conductors of line, that possess inductance. But, since the inductance of line in connection with the motion of the front of stress, also increases, in accordance with relationship (3.4), on it will be observed a voltage drop:

$$U = I \frac{\partial L(t)}{\partial t} = IvL_0 = v^2 U_0 C_0 L_0 \quad (4.1)$$

But a voltage drop across the conductors of line in the absolute value is equal to the stress, applied to its entrance; therefore in the last expression should be placed  $U = U_0$ . Then from relationship (4.1) we obtain

$$v = \frac{1}{\sqrt{L_0 C_0}}.$$

This relationship coincides with the velocity of propagation, obtained from the wave equations.

## 5. The Influence of the Rate of Lengthening the Long Line of on Its Input Resistance

Let us examine the long line, whose length can they will change with the mechanical method according to the law  $z = vt$ . Then capacity and inductance of line will change

according to the law

$$C(t) = zC_0 = vt C_0,$$

$$L(t) = zL_0 = vt L_0,$$

where  $C_0$  and  $L_0$  linear inductance and the capacity of line.

If we to the line connect the source of voltage  $U$ , thus it will charge the being increased capacity of line, for which from the source into the line there will be leak the current of

$$I = U \frac{dC(t)}{dt} = vUC_0 \quad (5.1)$$

This current there will be the leak through the conductors of line, that possess inductance. But, since the inductance of line increases on it will be observed a drop in the voltage of

$$U = I \frac{dL(t)}{dt} = vIL_0 = v^2 UC_0 L_0 \quad (5.2)$$

After dividing (5.2) on (5.1), we will obtain the effective resistance of the line

$$R(v) = vL_0 \quad (5.3)$$

If one considers that the velocity of propagation in the line is determined by the relationship

$$v_0 = \frac{1}{\sqrt{L_0 C_0}} \quad (5.4)$$

the effective resistance of line p can be expressed in fractions of this of the velocity

$$nv_0 \quad (5.5)$$

where the coefficient  $n$  is less than one.

Substituting (5.5) in (5.3) and taking into account (5.4), we obtain:

$$R(v) = n \sqrt{\frac{L_0}{C_0}} = nZ_0,$$

where  $Z_0$  is line characteristic.

If line is not enlarged, but it is reduced, then effective resistance becomes negative. This means that the line does not absorb energy, but returns it into the external circuits.

When line to be lengthened at the velocity greater than  $v_0$ , in this case line impedance is equally  $Z_0$ , since the pulse edge will not be able to overtake the end of the expanding line.

## 6. Conclusion

In the article are examined the laws of the self-induction of reactive elements. It is shown that such reactive elements as capacity and inductance with their connection to the current generators and voltage present the effective resistance, whose value depends on time. Is introduced the concept of the

parametric self-induction of reactive elements and long lines, which occurs when their parameters they change in the time. Introduction of this concept made possible to calculate the velocity of propagation of constant stresses and currents in the long lines, what cannot be made with the use of wave equations. Is examined the behavior of the input resistance of the long line, whose parameters change with mechanical method. It is shown that this resistance depends on the rate of lengthening line and can acquire negative importance in the case of the decrease of the length of line. It is shown also, that when the line is lengthened with the speed of larger than the signal velocity in it, this line has the input resistance, equal to the input resistance of infinite line.

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