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# Can the Principles of Relativistic Mechanics Have Direct Action in the Theory of Electromagnetism Surface Waves

A. V. Kukushkin<sup>1</sup>, A. A. Rukhadze<sup>2</sup>, F. F. Mende<sup>3</sup>

<sup>1</sup>Department of Computational Systems and Technologies, R. Alekseev Nizhny Novgorod State Technical University, Nizhny Novgorod, Russia

<sup>2</sup>A. Prokhorov General Physics Institute of the Russian Academy, Moscow, Russia

<sup>3</sup>B. Verkin Institute for Low Temperature Physics and Engineering NAS Ukraine, Kharkov, Ukraine

**Email address**

avkuku@gmail.com (A. V. Kukushkin), rukh@fpl.gpi.ru (A. A. Rukhadze),  
mende\_fedor@mail.ru (F. F. Mende)

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**Abstract**

It is shown that definition of antisymmetric tensor for the density of the energy-momentum of free electromagnetic field in the vacuum gives the possibility to write down the system of vector equations of motion for such mechanical attributes of field as the pulse densities and energy. Simple relativistic formula is obtained for the 3-vector of the speed of the energy density of the traveling waves. The concepts of the density of kinetic energy and rest energy in accordance with are introduced by the principles of relativistic mechanics for the slow waves with the speed of the energy, which does not reach the speed of light. The identification of these values as checking formula for the 3-vector of speed, they are carried out based on the example of heterogeneous longitudinal-transverse plane wave, which is used in the boundary-value problems of electrodynamics in the theory of surface waves. It is shown with the aid of the Lorenz's conversions that the rapid surface Tsennek's wave which could be extended above the plane interface vacuum - sea water, does not satisfy the requirements the special theory of relativity.

**1. Introduction**

The purpose of article is reduced to the attempt to solve the task about the determination of the 3-vector of the speed  $\vec{V}_E$  for the energy density of free electromagnetic waves in the vacuum with the use of the mathematical method, which was formulated in the article [1] in connection with to the motion of mass substance. The essence of method in [1] was reduced to the use in the tasks of the mechanics of antisymmetric tensor for the density of the energy-momentum of mass substance, continuous in Minkowski's space. The left side of the equation of motion for this substance, as it was shown in [1], it is obtained as a result reduction in the rank of the tensor with the aid of its internal multiplication (rolls) indicated with the 4-vector of the speed of substance. Use as the fundamental concept not of customary for all symmetrical tensor, but skew-symmetric tensor of the density of energy-momentum is here new for theoretical physics. As a result [1] for the substance, which occupies in the space the final volume (in particular, - point), was obtained relativistic "vortex equation of motion" (VEM), from which can be brought out two different, but equivalent forms for the equation of motion. Namely, if it is assumed that the

3-vector of the speed of material point clearly depends only on time, then VEM passes into the usual relativistic equation of motion Newtonian type [2]. But if it is assumed that it clearly depends only on three space coordinates, then it is obtained by VEM without the terms in the form of particular time derivatives. Specifically, this equation was integrated [1] in the classical limit of low speeds. In a test example of classical two-body problem the advantage of this approach was revealed. Namely, this method, in contrast to the methods, connected with the Newtonian equation, makes it possible to determine immediately all complete and everywhere dense families for all possible trajectories. All information about these families is placed in certain that determined from the method in [1]. The analytic complex variable function (material plane of motion it is substituted by equivalent complex plane). Thus, it is possible to assert that the mathematical approach, based on the use in the tasks of the mechanics of the antisymmetric tensor of the density of energy-momentum, has its advantages, at least, in the stationary case, which was examined [1] in the specific objectives of using this approach for the mechanics. The task, which is solved in this article, it is reduced to an attempt at the transfer of this method of the formulation of the tasks of mechanics to the tasks of electrodynamics. Motivation consists of the following. The field is had the characteristics of the mechanical system: the vector density function of pulse  $\vec{g}$  and the scalar density function of the energy  $W$ . However, in the theory of electromagnetism [2-4] is absent equation or the system of equations of motion, which mutually connect these two functions, from where it would be possible to define explicit expression for the 3-vector  $\vec{V}_E$ , of characteristic at the arbitrary point of Minkowski's space both the direction and the velocity of the displacement of the energy density of the field of free wave. To extract information about this of very Maxwell equations is impossible in principle, since these are mechanical equations and therefore they do not include 3-vector  $\vec{V}_E$  explicitly. Idea about the fact that this vector implicitly is present in the vector  $\vec{g}$ , for the mass-free substance, such as is electromagnetic field, it is in drastic contradiction with the special theory of relativity (SR). And although Umov's formula for the speed of energy widely is used in the applied electrodynamics [5, 6] from the point of view SR its use here it bears palliative nature. This is evident from the fact that for the calculations of the speed of energy, which give reasonable answers, into this formula must be substituted the certain averaged or integral values of the input there quantities [5, 6], while the speed of the energy density of field it must be the value, accurately determined at any arbitrary point of Minkowski's space. The fact that palliative solution of problem gives, nevertheless, reasonable results, held in control the development of the theory of the motion of energy of electromagnetic waves in the relativistic river bed. Finally, this badly affected the development of theory that it had also, until now it has negative consequences in the applications.

There is one clear example, which will be examined in the article, when this question has decisive importance for the

practice. In it the discussion will deal with the proof of possibility or impossibility of natural occurrence of rapid surface waves (SW), i.e., waves with the phase speed, which exceeds the speed of light in the vacuum  $c$ . Do possess similar waves physical sense it depends on that how is correlated the phase speed of simple harmonic wave  $V_{ph}$ , with the speed of the displacement of its energy. This question is, thus, the main thing for resolution of a question about the physical sense SW Tsennek's wave in which  $V_{ph} > c$  and it is rapid SW, hypothetically capable of freely being extended above the locally flat surface of sea (and of land), which would make it possible to use it in the systems of distant radio communication and in the radars of surface wave. Everything depends on that, is more or less  $c$  the velocity of propagation of energy density  $\vec{V}_E$  this rapid SW. Since this question was not raised in the scientific literature until most recently [7], first by this, probably, it is possible to explain that the fact that the scientific discussions about the physical sense of this wave last already more than hundred years and in no way they can end. Background of the question concerning SW Tsennek's wave is illuminated, for example, in [7] and we will not concern it. However, the last turn of this discussion arose in 2012, immediately after the appearance of an article [7], in which it was shown that the group velocity  $V_g$ , SW Tsennek's wave (but, therefore, also the speed of the displacement of its energy, since wave is weakly dissipative), as  $V_{ph}$ , it is more  $c$  and, therefore, it cannot be carried to the physical waves. However, is not agreeable the author of work which on [8], the basis of the use only of one palliative Umov's formula it attempts to prove with this that in the weakly-dissipative limit can occur the situation, when  $V_g > c$ , while  $|\vec{V}_E| < c$ . In other words, defending the wave of price list by any price, the author [8] asserts that in nature is a special case, when in the limit of vanishingly low losses of electromagnetic energy theorem M. Leontovich [5]:  $|\vec{V}_E| = V_g$ , it is not accurate. From this evidently, how a great significance for the practice has a question of correct relativistic, but not palliative, the determination of vector  $\vec{V}_E$  in the theory of electromagnetism.

Returning to our theme, let us note that in [1] it was noted that the 4-vector, formed from the values  $\vec{g}$  and  $c^{-1}W$ , from a formal point of view can be considered as the value, similar to 4-potential in the field theory, from where appears the possibility of determining the antisymmetric tensor for the density field of energy-momentum. Hence one step to the record of relativistic VEM, which includes value  $\vec{V}_E$  explicitly. All this was executed in [1] in connection with the influential substance. Now the same must be made for the mass-free substance, i.e., for the field with the infinite radius of action and to look, which will leave this. Indeed the energy density and pulse of electromagnetic field, in contrast to the same values for the influential substance, in the first place, of value being varied in the time and, in the second place, which is extremely important, for the mass-free field there does not

exist the associated frame of reference (AFR). However, for the last point there is an exception of in the form retarded waves. This, for example, slow SW, existence of which no one doubts. For them it is possible to indicate AFR. In accordance with by the principles of relativistic mechanics from this it follows that the functions for the density of kinetic energy and rest energy must be determined and identified for such waves. It is necessary to explain that this for the energies in connection with to wave processes? First, this, until now, it was not made, while need there exists, but, in the second place, on this must depend the formulation of equations of motion and generally very theory of the motion of energy of electromagnetic waves.

## 2. Equations of Motion for the Pulse Densities and Energy of Field Relativistic Formula for the 3-Vector $\vec{W}_E$

The mechanical attributes of field,  $\vec{g} = \sum_{i=1}^3 g_i \vec{e}_i$  and  $c^{-1}W$ , are united into the 4-vector of the energy density of the pulse [2]:

$$\vec{P} = \vec{g} - (W/c)\vec{e}_4, \tag{1}$$

$$\vec{g} = c^{-2}\vec{S}, \tag{2}$$

where  $\vec{S}$  is Poynting's vector [2],  $\vec{e}_i$  is the three-dimensional unit vectors of fixed Cartesian frame of reference (CFR) in Minkowski's space,  $\vec{e}_4$  is the corresponding temporary unit vector. Poynting into [5] established that for the arbitrary electromagnetic field in the vacuum, the values, entering in (8), are expressed as the vectors of field as follows:

$$\vec{S} = c[\vec{E} \times \vec{B}], \tag{3}$$

$$W = \frac{1}{2}(\vec{B}^2 + \vec{E}^2) = W_B + W_E. \tag{4}$$

Furthermore, for the identification of the value of the energy density of the rest of the retarded (surface) waves by us will be necessary expression for one of two 4-scalars of the field [3]:

$$\eta = \frac{1}{2}(\vec{B}^2 - \vec{E}^2) = W_B - W_E. \tag{5}$$

Let us note that in (3-5), as it is everywhere lower in similar cases, before the bracket in the right side of the formula is omitted the coefficient  $(4\pi)^{-1}$ . Generally speaking, the invariant of field (5) is calculated usually from the formula  $F_{\alpha\beta}F^{\alpha\beta}$  [3], where  $F_{\alpha\beta}$  is the tensor of field. It is clear that the invariant can be determined with an accuracy to material

constant,  $CF_{\alpha\beta}F^{\alpha\beta}$  with other one or sign or another. In the determination (5) as the constant accepted the number  $C = (8\pi)^{-1}$ , but with the equal success by this number could be and  $C = -(8\pi)^{-1}$ . Then 4- scalar would have a form:

$$\eta = W_E - W_B. \tag{6}$$

Essential is the fact that the record in the form (5) or (6) this invariant, in contrast to second 4-scalar  $\vec{E}\vec{B}$  [3], directly connects it with the concept of energy density. In (5) it is a difference in the energy densities of magnetic and electric field, while in (6) – vice versa. For us important is here the fact that the property of the invariance of 4-scalar (5) or (6) according to the sense can be used for the identification of the value of the energy density of the rest of wave. Actually, rest energy of any object – of material body or wave – must not depend on that, in which state is this object, in the state of rest or motion. In this sense, in the case of field, the property of the invariance of value (5) or (6) is necessary. For this 4-scalar  $\eta$  by us it will be necessary. From other side, value  $\eta$ , the defined by us as difference in the densities of corresponding energies of two pour on, so that it could be identified as energy density (rest), it must be the value of positive at any point of space and at any moment of time. For the retarded waves, depending on the polarization of wave, everywhere positive there can be either the value, determined in (5), or respectively in (6). For this very reason, in principle, for the determination  $\eta$  can be urgent both formulas, (5) and (6).

It is necessary to say that the main property of the invariance of 4-scalar (5) in the connection with the fact that it, in the essence, characterizes in the traveling wave the positive value of the scarcity of the energy density of its electric field in comparison with the energy density of its magnetic field, in physics, until now, how we know, application it did not find. For example, g. Beytmen in its known book [9] limited only to the fact that he used information about the numerical value  $\eta$  for the classification fields on as follows. He called fields, for which is satisfied the condition  $\eta \neq 0$ , by those not selfed-adjoin, and field, for which is satisfied the condition

$$\eta = 0 \tag{7}$$

by respectively self-adjointed. However, at present in the theory of electromagnetism a similar terminology is used in other sense. Therefore in order not to introduce confusion, we will call the fields, which g. Beytmen was called non-self-adjoint, longitudinal-transverse, and the fields, which satisfy condition (7) - by transverse, as this occurs actually. For the longitudinal-transverse waves (for example, SW) occurs the inequality:

$$|\vec{V}_E| < c \tag{8}$$

<sup>1</sup> Is possible to leave one determination (5), that we will make. Then, if  $\eta$  - everywhere negative function, then for the energy density of the rest of wave should be accepted value  $\eta$  with the opposite sign.

and we have respectively for the transverse,

$$|\vec{V}_E| = c. \tag{9}$$

For SR these are absolutely different situations. In the present work larger attention will be given to the first case. Actually, this case is special interest, since for such waves can be indicated AFR. Consequently, in the complete agreement with the principles of relativistic mechanics the density of the total energy of wave in CFR must consist of two terms:

$$W = W^{(k)} + W^{(r)}, (W^{(k,r)} > 0), \tag{10}$$

where  $W^{(k)}$  is the density function of kinetic wave energy, while  $W^{(r)}$  is the density function of its rest energy. That this for the energies, until it is unclear, but the principles of relativistic mechanics are universal, and wave energy cannot be exception. Is clear only one that for the case (8) there is moving hatching (subsequently – twice hatching) AFR, which is differed from all other moving inertial reference systems in terms of the fact that it moves in the direction of the motion of wave energy with speed  $V = |\vec{V}_E| < c$ , that it means that first term (10) in prime AFR is connected to neutral:

$$W^{(k)} = 0. \tag{11}$$

This energy density of rest is converted to AFR (with the use of Lorentz conversions) by means of the additional multiplication of expression (3) (undertaken with the correct sign) to the Lorentz factor. Rest energy of wave in some final volume in AFR taking into account relationship for the elements of volume,  $dV = \gamma^{-1}dV'$  (taking into account it is Lorentz the decrease of lengths), after the integration for it of energy density remains, thus, the same, what it was in the fixed frame of reference (CFR). This is required for the identification of rest energy of wave. It is obvious that a condition (11) according to its sense can be used for determining the numerical value of scalar  $|\vec{V}_E|$ . This is the most universal method of enumerating the speed of wave energy, since on the direction of the motion of the inertial reference systems of no limitations it is superimposed.

For the case (8) ( $\eta \neq 0$ ) the following relationships are important. Namely, if  $\eta \neq 0$ , then two formulas are equivalent to expression (3):

$$W_B = W_E + \eta. \tag{12}$$

Substituting them on the turn in (10), we will obtain the relationship, in one case,

$$W = 2W_E + \eta, W = E^2 + \eta, \tag{13}$$

other respectively

$$W = 2W_B - \eta, W = \vec{B}^2 - \eta, \tag{14}$$

Comparing (12) and (13) s (10), we conclude that two cases

are possible:

$$a) \eta > 0, W^{(k)} = 2W_E = \vec{E}^2, W^{(r)} = \eta. \tag{15}$$

$$b). \eta < 0, W^{(k)} = 2W_B = \vec{B}^2, W^{(r)} = -\eta. \tag{16}$$

Thus, for the case of longitudinal-transverse waves the principles of relativistic mechanics in the connection with the relativistic apparatus for electrodynamics make it possible to identify both the kinetic wave energy and its rest energy. The only necessary and second-order condition remains in this case requirement (i), that function  $\eta$  it must be fixed at any point of Minkowski's space. It must be either positive or negative at any point of Minkowski's space. Otherwise it is not possible to determine kinetic wave energy, since it is not possible to determine its rest energy. This requirement is extremely important for the theory of electromagnetism, since the principles of relativistic mechanics are general for physics.

Let us note that the case of purely transverse waves easily is obtained from the formulas as a result of the passage to the limit given above  $\eta \rightarrow 0$ . Actually, in this case we have  $W^{(r)} = 0$ ,

$$W^{(k)} = W, \tag{17}$$

$$W_B = W_E, (W_B^{(k)} = W_E^{(k)}). \tag{18}$$

In other words, kinetic wave energy is its total energy and, furthermore, in this energy there is no scarcity of its one component in comparison with another. The well known in physics symmetry of energies of electrical and magnetic field, characteristic for the traveling waves, occurs. However, this case in this work we examine will not in detail be, but let us only note that for the transverse waves the total energy of wave and kinetic – of concept synonymous. This absolutely not so for longitudinal-transverse wave of equations of motion for the energy density of the traveling waves does differ little in the mathematical aspect from the conclusion of Maxwell's equations in the vacuum. Therefore the details of conclusion for abridging the text we will omit, after noting only following.

Initial value undertakes the 4-vector (1), above which with an accuracy to sign 1 it is carried out the same operation, as in the electrodynamics [2, 3] above the 4-potential for obtaining the expression for the antisymmetric tensor (bivector) of the field:

$$\vec{T} = T_{\alpha\beta} = -\nabla \times \vec{P}, \tag{19}$$

where  $\nabla = \sum_{i=1}^4 \partial_i \vec{e}_i$ ,  $\partial_i = \partial/\partial x_i$ ,  $x_4 = ct$ . When deriving the

Maxwell equations in the vacuum they enter further thus. The tensor of field  $F_{\alpha\beta}$  they symbolically multiply internally with Hamilton's 4-operator  $\nabla$ . As a result the rank of tensor is reduced by one, and the new 4-vector, which they make level with zero, is formed. The first group of Maxwell's equations hence is obtained. The second group is obtained as a result the

external multiplication of tensor with Hamilton's operator (it searches for external derivative of tensor). Is formed the tensor of the third rank (3-vector 2), to which in Minkowski's space from a geometric point of view [2] of dualen the specific 4-vector. Equalizing it to zero, the second group of Maxwell's equations is obtained. The mathematical method of obtaining the relativistic equations of motion for the energy density of the traveling waves will differ from this only in terms of the fact that in the operations of reduction and respectively increase in the rank of tensor (20) is used not the operator Hamilton, but the 4- vector of the speed [1] of the substance <sup>2</sup>,

$$\bar{u} = \gamma(\bar{V}_E - c\bar{e}_4), \tag{20}$$

which in this case characterizes not the density of inert mass, but the density of kinetic wave energy. In other words, by energy density in (1) should be understood the density of kinetic wave energy,

$$\bar{P} = \bar{g} - (W^{(k)}/c)\bar{e}_4, \tag{21}$$

In fact, connecting up the operations indicated the tensor of the density of the energy-momentum of the moving wave with the 4-vector of its speed of energy (21), not simply there is no sense to include in 4-vector (1) the energy density of the rest of wave, completely indifferent to the reaction of the mechanical attributes of system to the process of motion, but this it would be large error. Here there must not be no reservations, and we still will return to discussion of this question and will prove this. Lowering ever more or less standard intermediate conversions, after using in them formula (2), let us extract the eventual result of in the form two new 4-vectors:

$$\dot{\bar{P}} = \gamma c^{-2} \left\{ \dot{\bar{\sigma}} - [\bar{V}_E \times \text{rot}\bar{S}] - c^{-1}(\bar{V}_E \cdot \dot{\bar{\sigma}})\bar{e}_4 \right\}, \tag{22}$$

$$\dot{\bar{U}} = \gamma c^{-2} \left\{ c^{-1} \cdot \text{rot}\bar{S} + c^{-1} [\bar{V}_E \times \dot{\bar{\sigma}}] - (\bar{V}_E \cdot \text{rot}\bar{S})\bar{e}_4 \right\}, \tag{23}$$

where

$$\dot{\bar{\sigma}} = \partial\bar{S}/\partial t + c^2 \text{grad}W^{(k)}. \tag{24}$$

Vector (22) (as (23) it possesses the physical dimensionality of temporary derivative (internal substantive time derivative of the appropriate tensor) from the pulse density (for this and it was necessary in the conversions to use not a standardized value of 4-speed). Consequently, relativistic equation of motion for a similar substance will come out, if we make level 4-vector (22) with the 4-vector of the density of the external forces, which act on the substance 3 (with the possible calculation of the forces of self-force). Since it is assumed that in the difference, for example, from gluon, electromagnetic field it is not charged, in the right side of the equation of motion it will stand zero. This there will be the equation,

which determines the energy-kinematics of field. However, its one it will be insufficient for the solution of the entire problem <sup>4</sup>. This equation should be supplemented, after making level with to zero 4-vectors (23). After this, as can be seen from expressions (22, 23), is formed the following complete system of vector equations in the 3d-space for determining the energy-kinematics of free wave pour on in the motion:

$$\dot{\bar{\sigma}} = [\bar{V}_E \times \text{rot}\bar{S}], \tag{25}$$

$$\bar{V}_E \cdot \dot{\bar{\sigma}} = 0, \tag{26}$$

$$c^2 \text{rot}\bar{S} = [\dot{\bar{\sigma}} \times \bar{V}_E], \tag{27}$$

$$\bar{V}_E \cdot \text{rot}\bar{S} = 0. \tag{28}$$

where the 3-vector  $\dot{\bar{\sigma}}$  is determined in (24).

Similarly, as Maxwell's equations they mutually connect the polar vector of electric field with the axial vector of magnetic field, system of equations (25) - (28) superimposes the specific connection on the analogous of vector, the being been mechanical attributes fields. These are polar vector  $\dot{\bar{\sigma}}$  and the axial vector  $\text{rot}\bar{S}$ . Furthermore, system includes explicitly the third vector  $\bar{V}_E$ , which, strictly, and the energy-kinematics of field is determined. Substantially important is the fact that the troika of the vectors in the 3d-space indicated forms the right-handed triad, which follows directly from the system (25) - (28). Actually, from (26) - (28) it follows that  $\dot{\bar{\sigma}} \perp \bar{V}_E$ ,  $\bar{V}_E \perp \text{rot}\bar{S}$ . But the remained two equations indicate that this is - the right-handed triad of mutually perpendicular vectors. Third equation does not be sufficient for the vectors of the right-handed triad, which, however, easily is established from first two (from the equations (25) and (27)):

$$\bar{V}_E = \frac{[\text{rot}\bar{S} \times \dot{\bar{\sigma}}]}{(\text{rot}\bar{S})^2}. \tag{29}$$

This is the simple relativistic formula, which determines at the arbitrary point of Minkowski's space the energy-kinematics of the free traveling waves. It is extremely important to give the comparative estimation of relativistic energy-kinematics with the energy-kinematics according to Umov's formula [11]. Formula

$$\bar{S} = \bar{V}_E W \tag{30}$$

it is in drastic contradiction with SR that shows the following reasoning, which we will conduct for simplicity for the case of transverse waves (see (7.9)). In this case we have a relationship (12) and therefore, if we take 4-divergence from (1) and to make level obtained scalar with zero, then taking

<sup>2</sup> Usually, with the account of mathematical apparatus SR 2, 3 use the value of dimensionless 4-speed, which is obtained from (21) by division on *c*

<sup>3</sup> The effectiveness of this method was proven and checked in 1 in connection with the influential substance.

<sup>4</sup> Exactly as in the electrodynamics the first group of Maxwell's equations without the second sufficient is not.

into account (2) we will obtain the scalar equation:

$$\text{div}\vec{S} + \partial W / \partial t = 0, \tag{31}$$

which is nothing else but the law of conservation of energy in the differential form for the field of the freely extended wave. Equation (31) has a form “of equation of continuity”. If equation (31) not only for the form, but also for the essence was “the equation of continuity”, then (30) it would be possible to consider formula relativistic. It would be derived from the conversions of Lorenz in the form convective formula, as it takes place with obtaining of analogous formula for the current density:

$$\vec{j} = \rho \vec{V}, \tag{32}$$

in which  $\rho$  is the charge density in CFR,  $\vec{V}$  is 3-vector of the speed of motion CFR relative to AFR, where the charge rests. The charge density is the electrodynamic characteristic of the influential substance, in which there is a nontrivial density of inert mass in AFR. For this substance there is AFR, in which the 4-vector of current density has only one projection on the temporary axis,  $\vec{J}' = -4\pi\rho'\vec{e}'_4$ . If we carry out the conversions of Lorenz for this 4-vector in order to write down it in CFR, then we will obtain the 4-vector

$$\vec{J} = 4\pi(c^{-1}\vec{j} - \rho\vec{e}_4), \tag{33}$$

in which are introduced the designations:  $\rho = \gamma\rho'$ ,  $\vec{j}$  is the 3-vector (33). Vector  $\vec{V}$  appeared in (32) and (33) only as a result of the use for the derivation of the formula (33) of the conversions of Lorenz. If we take 4-divergence from the 4-vector (34) and to equate the obtained expression with zero, then we will obtain “the equation of continuity”,  $\text{div}\vec{j} + \partial\rho/\partial t = 0$ , which is the same both on the form and in the content. Actually, formula (32) is obtained from the conversions of Lorenz. This is – relativistic formula and she indicates that the motion of electric charges is original cause for the phenomena, connected with the electric current.

Anything the similar in connection with to formulas (30) and (31) to say is generally cannot simply because for “the substance”, motion by which is subordinated to condition (9), it is not possible to indicate the associated frame of reference, where the 4- vector of the density of the energy-momentum of wave would have only one temporary projection. Therefore Umov's formula is not relativistic and is in drastic contradiction with SR, but “the equation of energy” (31), being relativistic, it is not “the equation of continuity”. Consequently, from a relativistic point of view formula (30) is erroneous. The of course specific nonjoining of formulas (30) and (31) in the electrodynamics did not remain unnoticed and therefore in the applied electrodynamics is used not formula (31), and its simplified analog:

$$V_E^{(x)} = \vec{S}_x / \vec{W}, \tag{34}$$

where  $\vec{S}_x$  is energy flow in the direction of the axis  $Ox$  for the unit of time through a certain area (for example, through the cross section of waveguide [6]),  $\vec{W}$  is total energy of wave, accumulated in the appropriate volume for the unit of time (during the oscillatory period for the simple harmonic waves). We still will return to a question of the comparison of two forms of the energy-kinematics of waves via the comparison of the results of determining of vector  $\vec{V}_E$  or its length, and also its projections, by two methods: according to the formulas (29) and (34).

### 3. Energy-Kinematics of Longitudinal-Transverse Waves in the Vacuum Surface Waves Formula (29)

In Fig. 1 are shown two Cartesian coordinate systems. Primed system is obtained from the latter to the angle of  $\psi$  not touched by rotation clockwise. The coordinates of two systems are connected together with system of equations:

$$\begin{cases} x' = x \cos \psi - y \sin \psi, \\ y' = x \sin \psi + y \cos \psi, \\ z' = z. \end{cases} \tag{35}$$

The axis  $z$  is perpendicular toward the plane of drawing and is directed toward the reader. The connection between the appropriate unit vectors of two systems is expressed just as in (35) between the coordinates.

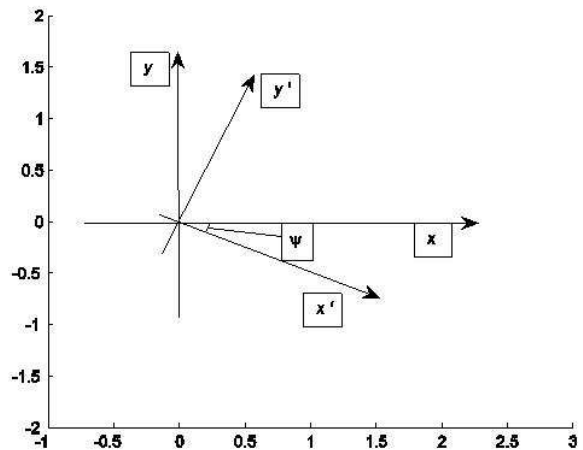


Fig. 1. Two Cartesian coordinate systems in the fixed frame of reference. Paul SW does not depend on coordinate  $Z'$  ( $z$ ), which it is perpendicular toward the plane of drawing and it is directed toward the reader. Paul SW exponentially it diminishes in the direction of an increase in the transverse coordinate  $y'$ . Non-dissipative slow SW runs in the direction of the growth of the longitudinal coordinate  $x'$ . Dissipative SW runs in the direction of the growth of the untouched longitudinal coordinate  $x$ .

Let assuming that now along the axis  $Ox'$  in this figure to the side of the build-up of the longitudinal coordinate of in the vacuum runs harmonic nondamping longitudinal-transverse N

SW, the components of field of which for TM polarized waves with the longitudinal component of electric field are written as follows:

$$\begin{Bmatrix} B_{z'} \\ E_{y'} \end{Bmatrix} = \begin{Bmatrix} 1 \\ \partial\beta'/\partial x' \end{Bmatrix} \cos(k_0\beta') \exp(-k_0\delta'), \quad (36)$$

$$E_{x'} = (\partial\delta'/\partial y') \sin(k_0\beta') \exp(-k_0\delta'), \quad (37)$$

$$\beta' = x'(\partial\beta'/\partial x') - ct = x' \cosh \zeta - ct, \quad (38)$$

$$\delta' = y'(\partial\delta'/\partial y') = y' \sinh \zeta, \quad (39)$$

$k_0 = \omega/c$ ,  $\partial\beta'/\partial x' = \cosh \zeta$  is the standardized longitudinal wave number of wave or its moderating ratio,  $\partial\delta'/\partial y' = \sinh \zeta$  is the standardized transverse wave number,  $\zeta \in [0, \infty]$ . In the limit  $\zeta \rightarrow 0$  the field NSW passes into the cross field of uniform plane wave. In contrast to this limiting case the amplitude of field NSW as the scalar density function of wave energy, on the plane of equal phases, which coincides with the coordinate plane  $y'Oz'$ , is distributed exponentially. The heterogeneity of the field of wave on the plane of equal phases is the guarantee of the fact that its energy-kinematics is accurately determined by formula (29), since, in this case the condition is satisfied (ii):  $\text{rot}\vec{S} \neq 0$  (see the formula (29)). It also follows from (38) and (39) that

$$(\partial\beta'/\partial x')^2 - (\partial\delta'/\partial y')^2 = 1. \quad (40)$$

Relationship (40) indicates that, as must be, the field of wave satisfies the wave equation (transverse wave number it is imaginary). However, with  $y' \rightarrow -\infty$  the value of the field of wave it unlimitedly grows exponentially, therefore, if vacuum occupies entire space, then there is no sense to investigate this solution of Maxwell's equations. Sense appears, if one assumes that lower half-space,  $y' < 0$ , is filled with uniform cold plasma [7], dielectric constant of which – value real and negative. In this case the plane interface “vacuum- plasma” possesses property to direct along the boundary  $y' = 0$  of slow SW [7], field of which in the space above the boundary,  $y' > 0$ , is described by the formulas (36) - (40) [7].

Is as shown higher, the first, that it is necessary to make for determining the relativistic energy-kinematics of this wave in the vacuum (in the space above the boundary) this to identify the density function of kinetic wave energy, it is for which necessary to first determine the density function of rest energy or, differently, it is simple to calculate 4-scalar (5) and to look, it does satisfy condition necessary for this (i)? Calculations we conduct in the prime system of coordinates, Fig. 1. Using designation,

$$\varepsilon = \cos^2(k_0\beta'), \left( \varepsilon = \frac{1}{2} [1 + \cos(2k_0\beta')] \right), \quad (41)$$

we will obtain from (36) the formula:

$$W'_B = \frac{1}{2} B_{z'}^2 = \frac{1}{2} \bar{B}^2 = \frac{1}{2} \varepsilon \exp(-2k_0\delta'). \quad (42)$$

Function for the density of electrical energy of wave in accordance with the formulas (36) (37) will take the form:

$$W'_E = \frac{1}{2} [E_{y'}^2 + E_{x'}^2] = \frac{1}{2} \bar{E}^2 = \frac{1}{2} [(\partial\beta'/\partial x')^2 \varepsilon + (\partial\delta'/\partial y')^2 (1 - \varepsilon)] Q,$$

where for the reduction of the record of formulas is introduced the designation:

$$Q = \exp(-2k_0\delta'). \quad (43)$$

and value  $\delta'$ , linearly depending on the coordinate  $y'$ , is determined in (39). Taking into account (40) we come hence to the following expression:

$$W'_E = \frac{1}{2} [\varepsilon + (\partial\delta'/\partial y')^2] Q. \quad (44)$$

Reading from (42) expression (44), we find 4-scalar :

$$\eta = -\frac{1}{2} (\partial\delta'/\partial y')^2 Q = -\eta_0 Q. \quad (45)$$

Taking into account (39) and (43) from (45) we conclude that for this wave  $\eta < 0$  at the arbitrary point of the space, where is determined its field, and, therefore, condition (i) is satisfied<sup>5</sup>. We can identify the energy densities of rest and kinetic wave energy in CFR as follows (see (17)):

$$W'^{(r)} = -\eta = \eta_0 Q, \quad (46)$$

$$W'^{(k)} = 2W'_B = \bar{B}^2 = \varepsilon Q. \quad (47)$$

From (47) it follows that for TM polarized SW the total kinetic wave energy is equal to the doubled value of magnetic energy, i.e., rest energy of wave is not contained in the energy of magnetic field. Or, otherwise, magnetic energy of wave composes exactly half of its total kinetic energy. The second-half of kinetic energy, obviously, is contained in the electric field energy. All this can be expressed by the formula:

$$W'_B = W'^{(k)} = \frac{1}{2} \varepsilon Q. \quad (48)$$

Actually, formula (44) taking into account relationships (46) - (48) can be written down thus:

$$W'_E = W'^{(k)} + W'^{(r)}, \quad (49)$$

where

$$W'^{(k)} = \frac{1}{2} \varepsilon Q. \quad (50)$$

<sup>5</sup> For TE polarized SW, as it is possible easily to surmise, condition (i) also is satisfied, but  $\eta > 0$ .

We conclude from (48) - (50) the formulas that moving, kinetic wave energy consists of two equal in magnitude components: kinetic energies of electrical and magnetic field. The common for physics symmetry of energies here preserves its value, but only minus rest energy, which for TM polarized SW is contained in the electric field energy, and for TE polarized SW – in the energy of magnetic field.

As it follows from the formulas (43), (46) and (39), the energy density of the rest of wave is the density of electrical energy of permanent field, which, of course, and cannot be moved regarding. Therefore for describing the motion of wave, where the operation of reduction in the rank of the antisymmetric tensor of the density of its energy-momentum with the aid of its internal multiplication with the 4-vector of speed enters, the start in this tensor of the value of the energy density of the rest of wave would be large error. At the same time, to consider that rest energy of wave remains on the spot, while kinetic energy, after being separated from it, is moved, it is also cannot. Rest energy is moved together with the kinetic energy on the strength of the fact that it is moved the volume, in which are concluded both parts of the energy. Thus, there is no contradiction here, since, in one case the discussion deals with the displacement of energy density, while in other with the motion of the energy 6.

After the made observations we pass to the calculations. Subsequently with the calculations of the values, entering formula (29), will be required expression for the derivatives of the function  $\epsilon$ , determined in (41). Let us introduce the following designations for the appropriate derivatives:

$$\frac{\partial \epsilon}{\partial(k_0\beta')} = \epsilon', \quad \frac{\partial^2 \epsilon}{\partial(k_0\beta')^2} = \epsilon'', \quad \frac{\partial \epsilon}{\partial t} = -\omega \epsilon'. \quad (51)$$

Furthermore, in the intermediate mathematical conversions will be used relationship,

$$\epsilon''/2 = 1 - 2\epsilon, \quad (52)$$

escaping from determination for the function  $\epsilon$  in (56). With the calculation of Poynting's vector by us will be necessary also escaping from (56) and (50) the relationship:

$$\epsilon'/2 = -\sin(k_0\beta') \cos(k_0\beta'), \quad (\epsilon' = -\sin(2k_0\beta')). \quad (53)$$

Knowing expressions for the vectors of field (see (36), (37)), it is not difficult to calculate Poynting's vector in CFR:

$$\begin{aligned} \vec{S}' &= c \left[ (\partial\beta'/\partial x') \epsilon \vec{e}_{x'} + (\partial\delta'/\partial y') (\epsilon'/2) \vec{e}_{y'} \right] Q \\ &= (s_{x'} \vec{e}_{x'} + s_{y'} \vec{e}_{y'}) Q \end{aligned} \quad (54)$$

Taking into account the third formula from (51) we differentiate this expression on the time and we will obtain

expression,

$$\dot{\vec{S}}'/\partial t = -\omega c \left[ (\partial\beta'/\partial x') \epsilon' \vec{e}_{x'} + (\partial\delta'/\partial y') (1 - 2\epsilon) \vec{e}_{y'} \right] Q, \quad (55)$$

with conclusion of which was used relationship (51). let us calculate entering the vector gradient from the function of , determined in (47):

$$c^2 \text{grad} W^{(k)} = \omega c \left[ (\partial\beta'/\partial x') \epsilon' \vec{e}_{x'} - 2\epsilon (\partial\delta'/\partial y') \vec{e}_{y'} \right] Q. \quad (56)$$

Summarizing (55) and (56) by formula (25), we find:

$$\dot{\vec{\sigma}}' = -\omega c (\partial\delta'/\partial y') Q \vec{e}_{y'}. \quad (57)$$

Let us note one important property (j) of the executed conversions. Namely, both vectors, (55) and (56), have a dependence from both of coordinates and time. However, in the vector  $\dot{\vec{\sigma}}'$  after addition (55) (56) remains dependence as function  $Q$  only of one transverse coordinate  $y'$  (Fig. 1). Axial vector is determined so  $y'$  (Fig. 1):

$$\text{rot} \vec{S}' = (\partial S_{y'}/\partial x' - \partial S_{x'}/\partial y') \vec{e}_{z'}. \quad (58)$$

The components of Poynting's vector entering here are determined in (54), and particular derivatives of them are calculated from the formulas:

$$\frac{\partial S_{y'}}{\partial x'} = \omega \frac{\partial\delta'}{\partial y'} \frac{\partial\beta'}{\partial x'} \frac{\epsilon''}{2} Q, \quad \frac{\partial S_{x'}}{\partial y'} = -\omega \frac{\partial\delta'}{\partial y'} \frac{\partial\beta'}{\partial x'} 2\epsilon Q. \quad (59)$$

Therefore, taking into account relationship (52) from (58) (59) we will obtain finally,

$$\text{rot} \vec{S}' = \omega (\partial\delta'/\partial y') (\partial\beta'/\partial x') Q \vec{e}_{z'}, \quad (60)$$

$$(\text{rot} \vec{S}')^2 = \omega^2 [(\partial\delta'/\partial y') (\partial\beta'/\partial x')]^2 Q^2. \quad (61)$$

Particular derivatives of the components of Poynting's vector in (59) possess dependences from both of coordinates and time, but in the vector  $\text{rot} \vec{S}'$  there remains only one dependence (in the value  $Q$ ) from the transverse coordinate  $y'$ . The noted property (j), thus, realizes because of the linear combination of two derivatives in (58). This realizes because of the fact that in the calculations the value is considered  $S_{y'}$ . The integral part of the calculations is connected with it, despite the fact that, as can be seen from (53) and (54), value average during the oscillatory period  $S_{y'}$  is equal to zero, (jj)  $\bar{S}_{y'} = 0$ . We specially emphasize this because in the standard calculations of speed according to Umov's formula, as is well known [5] (see it is below), value  $S_{y'}$  they intentionally throw out from the examination on the basis of the property (jj). As if it gives the right to ignore the inherent property of wave, which they escape from the structure of Maxwell's equations.

From (57) and (60) taking into account, that  $\vec{e}_{z'} \times \vec{e}_{y'} = \vec{e}_{x'}$ ,

<sup>6</sup> In the reality is moved not the simple harmonic wave, but the wave packet, which occupies in the space a certain final volume, but the laws of the motion of the energy density of simple harmonic wave in view of existence of Fourier's theorem about the idea of impulse functions by Fourier integral are the most important element of scientific "interface".



we find the desired vector, which, because of the property (j), functionally depends only on  $y'$ :

$$[\text{rot}\vec{S}' \times \vec{\sigma}'] = \omega^2 c (\partial\delta'/\partial y')^2 (\partial\beta'/\partial x') Q^2 \vec{e}_{x'}. \quad (62)$$

Both vector (62), and scalar (61), into which is divided this vector in formula (29), both values they have by coefficient a function  $Q^2$ , which with the division it is reduced and therefore fraction loses existing in the numerator and the denominator the last remained functional dependence on the transverse coordinate  $y'$ . Thus, as a result the division of numerator into the denominator in the formula (29) is formed the sliding 3-vector:

$$\vec{V}_E = c (\partial\beta'/\partial x')^{-1} \vec{e}_{x'} = (c/\cosh \zeta) \vec{e}_{x'}, \quad (63)$$

which at any point of space and at any moment of time has one and the same length and one and the same direction, as must be for so simple a solution of the Maxwell's equations as NSW. It is completely obvious that if we throw out from the calculations the value  $S_{y'}$  on the basis of its property (jj), the this it did not happen, and the energy-kinematics of this simple wave would remain completely indeterminate. Let us recall that  $\cosh \zeta$  - this is the moderating ratio of wave. Therefore the module the velocity vector of the energy density of wave is equal to phase speed,

$$|\vec{V}_E| = V_{ph} < c. \quad (64)$$

This is regular for the simple harmonic wave, since. the sine wave of energy density does not have envelope, which with the presence in system of dispersion would be extended some by another, for example, group velocity.

Let us look now, what answer we would teach, after including in the expression for the function of moving energy density of the retarded wave the density function of its rest energy, thus, after destroying the bases of relativistic mechanics in connection with to the description of the motion of wave energy. For this in all formulas it is necessary to conduct the replacement:  $W^{(k)} \rightarrow W'$ , for which, summarizing expressions (42) (44), we will obtain:

$$W' = \left[ \varepsilon + \frac{1}{2} (\partial\delta'/\partial y')^2 \right] Q. \quad (65)$$

Changes will, first of all, concern the formula (56). New result taking into account (65) will here be the following:

$$c^2 \text{grad} W' = \omega c \left\{ (\partial\beta'/\partial x') \varepsilon \vec{e}_{x'} - 2 (\partial\delta'/\partial y') \left[ \varepsilon + \frac{1}{2} (\partial\delta'/\partial y')^2 \right] \vec{e}_{y'} \right\} Q.$$

Respectively will change the formula (57),

$$\vec{\sigma}' = -\omega c \left\{ (\partial\delta'/\partial y') \left[ 1 + (\partial\delta'/\partial y')^2 \right] \right\} Q \vec{e}_{y'},$$

but property (j) remains. Taking into account (40) we simplify this formula to the form:

$$\vec{\sigma}' = -\omega c \left\{ (\partial\delta'/\partial y') (\partial\beta'/\partial x')^2 \right\} Q \vec{e}_{y'}.$$

Following this will change the formula (62):

$$[\text{rot}\vec{S}' \times \vec{\sigma}'] = \omega^2 c (\partial\delta'/\partial y')^2 (\partial\beta'/\partial x')^3 Q^2 \vec{e}_{x'}. \quad (66)$$

After substituting formula (66) and (61) in (29), we will obtain the following answer:

$$\vec{V}_E = c (\partial\beta'/\partial x') \vec{e}_{x'} = (c \cdot \cosh \zeta) \vec{e}_{x'}, \quad |\vec{V}_E| > c.$$

It shows that the negligence by the rules of relativistic mechanics in the theory of electromagnetism leads to blunder in energy-kinematics. Let us examine questions of energy-kinematics on the basis of the use for these purposes of Umov's formula (30),

$$\vec{V}_E = S'/W', \quad (\vec{V}_E = \vec{S}'/\vec{W}'), \quad (67)$$

Recorded in the prime frame of reference, Fig. 1. In the denominator of formula (68) it stands, as this is everywhere accepted [5, 6], the density function of the total energy of wave, determined by expression (65), in the numerator 3-vector (54). Taking into account expressions (41)-(40) formula (65) can be written down thus:

$$W' = \frac{1}{2} \left[ (\partial\beta'/\partial x')^2 + \cos(2k_0\beta') \right] Q. \quad (68)$$

With the substitution (68) and (54) in (67) the function  $Q$  they are reduced, but also only. Property noted earlier (j) or similar to it these calculations do not have. In numerator and denominator of fraction irreducible dependences on the longitudinal coordinate and the time remain. The energy-kinematics of the waves by Umov's formula (30), therefore, is not described for reasons, which are indicated above. The relationship between the vector of the energy current density and moving by energy density is not convective. However, a palliative way out situation lies in the fact that into numerator and denominator of Umov's formula are substituted averages during the period of oscillations (or, otherwise, average on a certain area and respectively in the volume, limited by this area) of the value of the energy current density and wave energy. It follows from (42) and (54)  $\bar{\varepsilon} = 1/2$ ,  $\bar{\varepsilon}' = 0$ , and we find from (54) the formula:

$$\vec{S}' = \frac{c}{2} (\partial\beta'/\partial x') Q \vec{e}_{x'}. \quad \text{Accordingly from (68) it follows that in}$$

this case  $\vec{W}' = \frac{1}{2} (\partial\beta'/\partial x')^2 Q$ . After substituting into Umov's formula, which is concluded  $v$  (67) in the parentheses, indicated average values, we will obtain expression for the vector  $\vec{V}_E$ , coinciding with its relativistic value in the formula (63). Probably, to this it is possible to give some explanation in that plan, that error in the denominator (it is taken total energy instead of the kinetic) it is compensated by error in the numerator (it is ignored the being varied value of the cross flow of energy). In any event, but the heuristic formula of

Umov for the average values gives correct answer. Regardless of the fact, as this it is possible to explain, between the appropriate average values for the density of total energy and its flow realizes convective connection. By this is explained the success in the applied electrodynamics of the heuristic Umov's formula, which, however, is in rough contradiction with bases SR. Probably, despite not what, this very useful property of Umov's formula will still for long, also, by the inertia support its urgency in the applications, but there is one extended error in the practice of its application, to which we will focus attention below in connection with the discussion along Tsennek's wave.

#### 4. Conversions of Lorenz and the Energy-Kinematics of the Retarded Waves

The more universal method of determining the velocity  $V_E$  energy density in that indicated, predetermined, direction consists of the use for these purposes of conversions of Lorenz and formula (11), of based on the principles relativistic mechanics.

Actually, from those calculated in the fixed prime frame of reference (see Fig. 1) the values of the Poynting vector and energy density it is possible to compose the 4-vector of the density of energy-momentum and further problem is solved simply. Namely, it is necessary to convert this 4-vector to the associated twice prime frame of reference, which moves with the speed  $v < c$  in parallel to the fixed axis  $Ox'$  in the direction of the growth of the coordinate  $x'$ . Frame of reference will accompany the motion of energy when and only when the speed of forward motion of the frame of reference  $v$  it will be equaled with the speed of wave energy in this direction  $V_E$ . The condition of the equality of the speeds indicated, is equivalent to the requirement of equality zero kinetic composing wave energies  $W^{(k)} = 0$ , in the twice prime moving coordinate system,

Thus, the velocity of propagation of wave energy  $V_E$  in the arbitrarily selected direction (in particular, in the direction, parallel with respect to the longitudinal axis  $Ox'$ , moving in which wave does not experience damping), it is determined from the one-only condition, - the equation (11). Let us write down expression for this 4-vector:

$$\vec{P}' = c^{-2} [s_x \vec{e}'_1 + s_y \vec{e}'_2 - c(\epsilon + \eta_0) \vec{e}'_4] Q, \tag{69}$$

where, according to (54),

$$s_x = c(\partial\beta'/\partial x')\epsilon, \tag{70}$$

$$s_y = c/2(\partial\delta'/\partial y')\epsilon' \tag{71}$$

the function  $\epsilon$  and  $\epsilon'$  are determined respectively in (41) and (53), and  $\eta_0$ , determined in (69), according to (45), there is simply the constant:

$$\eta_0 = \frac{1}{2}(\partial\delta'/\partial y')^2 = \frac{1}{2}\sinh^2 \zeta. \tag{72}$$

Moreover, the function  $\epsilon Q$ , as this follows from (65), exactly and corresponds to kinetic component of energy. This must be considered when deriving the equation (11).

The Lorenz's conversions for the coordinates take following form [2, 3]:

$$\begin{aligned} x' &= \gamma(x'' + vt''), \\ ct &= \gamma[ct'' + (v/c)x'']. \end{aligned} \tag{73}$$

Unit vectors are converted just as coordinates,

$$\begin{aligned} \vec{e}'_1 &= \gamma(\vec{e}''_1 + (v/c)\vec{e}''_4), \\ \vec{e}'_4 &= \gamma[\vec{e}''_4 + (v/c)\vec{e}''_1]. \end{aligned} \tag{74}$$

The remained two space coordinates  $(z, y')$  and their unit vectors in this case experience identity transformations and we them, therefore, we lower. After substituting (70), (71) and (74) in (69) and after grouping similar terms, we will obtain expression for 4-vector (69) in the twice prime moving frame of reference,

$$\vec{P}'' = c^{-2} [s''_1 \vec{e}''_1 + s''_2 \vec{e}''_2 - cw'' \vec{e}''_4] \exp(-2k_0\delta''), \tag{75}$$

$$s''_1 = \gamma[(s_x - v\epsilon) - v\eta_0], \quad s''_2 = s_y, \tag{76}$$

$$w'' = \gamma[\eta_0 - (s_x v/c^2 - \epsilon)].$$

After substituting into these last expressions of formula (70) and (71), we will obtain

$$s''_1 = c\gamma[\epsilon(\partial\beta'/\partial x' - v/c) - \eta_0(v/c)], \tag{77}$$

$$w'' = \gamma[\eta_0 - (\partial\beta'/\partial x'(v/c) - 1)\epsilon].$$

Taking into account everything said higher, we conclude that condition (11) will be satisfied, if we the numerical coefficient before  $\epsilon$  in the last expression make level with zero,

$$v = c(\partial\beta'/\partial x')^{-1} = c/\cosh \zeta, \tag{78}$$

from where we finally find that

$$w'' = \gamma\eta_0, \tag{79}$$

and since regarding we have  $v = V_E$ ,  $V_E = c/\cosh \zeta$ , which is in the complete agreement with expression (63), found with another method, namely, with the use of vector formula (29). Further, from (78) we find that

$$v/c = (\partial\beta'/\partial x')^{-1} \tag{80}$$

from where we have taking into account relationship (40)

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{\partial\beta'/\partial x'}{\partial\delta'/\partial y'} = \frac{\cosh \zeta}{\sinh \zeta} \quad (81)$$

it is possible to write down

$$\gamma \left( \frac{\partial\beta'}{\partial x'} - v/c \right) = \frac{\partial\delta'}{\partial y'}, \quad \gamma_0 = \frac{1}{2} \frac{\partial\beta'}{\partial x'} \frac{\partial\delta'}{\partial y'}, \quad \gamma_0 (v/c) = \frac{1}{2} \frac{\partial\delta'}{\partial y'} \quad (82)$$

After substituting the first and third of formulas (87) in (77), we find taking into account determination (41) that

$$s_1'' = c(\partial\delta'/\partial y')(\epsilon - 1/2) = c/2(\partial\delta'/\partial y') \cos(2k_0\beta''), \quad (83)$$

Further, according to formulas (76), (71) and (41), we have

$$s_2'' = -c/2(\partial\delta'/\partial y') \sin(2k_0\beta''), \quad (84)$$

$$\vec{P}'' = \frac{1}{2c} \left( \frac{\partial\delta'}{\partial y'} \right) \left[ \cos \left( 2k_0 \frac{\partial\delta'}{\partial y'} x'' \right) e_1'' - \sin \left( 2k_0 \frac{\partial\delta'}{\partial y'} x'' \right) e_2'' - \frac{\partial\beta'}{\partial x'} e_4'' \right] \exp(-2k_0\delta''), \quad (87)$$

where  $\delta'' = \delta' = (\partial\delta'/\partial y') y'' = y'' \sinh \zeta$ .

It is evident from (87) the formula that the energy density of rest in obtained AFR does not depend on time. Therefore the equation of energy balance (31) in this frame of reference takes the form:  $\text{div} \vec{S}'' = 0$  It is easy to see that this equation is satisfied. Other it could not be, since. the three-dimensional components of 4-vector (69), which went through transformations of Lorenz, are the components of Poynting's vector in the fixed frame of reference, and the density function of the total energy of wave is undertaken as the temporary component in (69). So it will be always, namely, in any AFR. This is very important, since. for the dissipative case the energy density of the rest of wave in appropriate AFR, which is differed from that found above both from the direction and from the speed, because of the dissipation will already depend on time. But the equation of energy balance will be satisfied automatically for the same reason, and to check this already there is no need.

The latter, that it is necessary to note, this that the fact that in contrast to the influential substance, the pulse density of the field of the retarded longitudinal-transverse wave (here this SW) is by no means equal to zero AFR, but this vector function with two components is sinusoidal static on the longitudinal coordinate, i.e., thickened on the spot, "frozen" standing wave. Zero equally average value of pulse density in the three-dimensional section, equal to the length "of wave". In CFR zero average value possessed only transverse component. Everything this bears out the fact that and in the case of the longitudinal-transverse waves, the velocity of propagation of energy of which less  $c$ , in the 4-vector of the density of energy-momentum three-dimensional components in AFR exist. This is one additional proof of the fact that from a relativistic point of view the convective formula of Umov (30) is the certain phantom, which successfully continues its existence in the electrodynamics, after being reached by it as the inheritance from the naive ideas of prerelativistic physics. The same phantom is rapid SW

and also, if we consider the second formula in (82) and formula (79), it is possible to write down,

$$w'' = 1/2(\partial\beta'/\partial x')(\partial\delta'/\partial y'). \quad (85)$$

Into the argument of trigonometric functions from (83) and (84) enters new variable  $\beta''$ , which is found from the old variable  $\beta'$ , determined in (38), after its conversion with the aid of the Lorentz equations (74) for the coordinates. If we substitute (73) in (38), then taking into account (78) and formulas (82) we find,

$$\beta'' = (\partial\delta'/\partial y') x'' = x'' \sinh \zeta. \quad (86)$$

After substituting formulas (83) - (86) in (75), we find finally,

Tsennek's wave. This is weakly dissipative wave; therefore in conclusion it is necessary to examine the case of dissipative SW. But before this let us make one additional observation apropos of energy-kinematics retarded SW. the latter, that it is necessary to say, this that the fact that, as can be seen from formula (87), in the limit  $\delta'' = \delta' = (\partial\delta'/\partial y') y'' = y'' \sinh \zeta$ , with which the field SW is approached field OSW, and  $v \rightarrow c$ , the 4- vector of the density of energy-momentum is fixed to zero,  $\vec{P}'' \rightarrow 0$ , since is fixed to zero common factor, which stands in expression (87) before the bracket,  $(\partial\delta'/\partial y') \rightarrow 0$ . Let us recall that this standardized transverse wave number SW. This fact,  $\vec{P}'' \rightarrow 0$  testifies that the energy-kinematics of wave becomes indeterminate. Reason is the fact that in CFR, as is evident (60), in this limit we have also:  $\text{rot} \vec{S}' \rightarrow 0$ . This the special case of passage from is longitudinal - transverse the waves to the purely transverse, on what we here dwell will not be. Let us note only that the result  $\vec{P}'' \rightarrow 0$  is symptomatic. It is worthwhile only to glance at the system of equations of motion (25) - (28), from one side, and to the formula (81), where  $\gamma \rightarrow \infty$ , with another. The first attests to the fact that from the equations of motion in CFR falls out the 3-vector  $\vec{V}_E$ , and the second about the fact that the method of the conversions of Lorenz, ceases to work, since. in this limit it cannot be determined AFR, in consequence of which equation (11) becomes not urgent.

## 5. Conversions of Lorenz for Dissipative SW End of the Discussion on the Tsennek's Wave

For the passage to the case of dissipative SW lower half-space in Fig. 1 ( $y < 0$ ) we will consider as the now filled dissipative homogeneous plasm-like medium, in particular, - by sea water. Upper half-space ( $y > 0$ ) is filled as before with

vacuum. In it inclined toward the interface of two media ( $y = 0$ ) in the direction of the axis of it is extended without damping of TM polarized longitudinal-transverse sinusoidal retarded SW. Everything, which relates to this wave, was examined above, and this remains without the changes. But now the conditions of task changed. Non-dissipative slow SW indicated falls inclined on the boundary of two media. Therefore in the direction of axis  $Ox$  field this SW will experience damping. This immanent property of wave is convenient to consider it physically caused by dissipation in the lower medium. If is solved boundary-value problem for the SW dissipative wave On the Border  $y = 0$  and its complex wave numbers are determined according to the method of complex amplitudes, then two well-defined real numbers correspond to them: moderating ratio non-dissipative SW  $\partial\beta'/\partial x' = \cosh \zeta$  the angle of its incidence  $\psi$  (see Fig. 1) to the boundary. The corresponding boundary-value problem  $y = 0$  was On the Border accurately solved into 7, the complex wave numbers in wave in the vacuum (and in the lower medium) – longitudinal and transverse – they are determined by final analytical expressions in the radicals, which was, of course, known and earlier, even before the appearance of the work [7]. Let us in passing note that, apparently, the author of work [7] with this is not agreeable. Otherwise as then to explain that during the solution of this simple boundary-value problem it does come running to the expansion of radicals from the dispersion equation in Taylor series? Retaining its dominant terms, it obtains the, thus, approximate solution of dispersion task, while in it there is the simple and precise analytical solution. But this not a central failure in the work [8], in it are more serious errors, which bear fundamental nature, and about them it will be said below. Thus, with the presence of dissipation in the lower medium  $\psi \neq 0$ . In this case the field of TM polarized SW in the vacuum under specific conditions, which were revealed in [7], it is field SW and, in particular, the field of the rapid Tsennek's wave (lower medium – sea water). The wave can be both slow and rapid (Tsennek's wave) dissipative SW, everything depends on the properties of plasma-like medium, which fills lower half-space [7].

From a formal mathematical point of view rapid SW anything not worse than the slow: as the first, the second satisfy boundary condition at transverse infinity. The position of the author is clear from this point [8] of view, it attempts to preserve this wave in the discrete spectrum of the waves of the open waveguide [10] by any price. For achievement this purpose, as it was said at first, it even asserts in [8], that the weakly dissipative Tsennek's wave this the special case in the science, because the conflicting inequalities realize in spite of the common sense for it: (k)  $V_g > c$ , (h)  $V_E < c$ . And also, over that, in spite of vanishingly low losses, common for this limit method of slight disturbances, as asserts the author in [8] it does not work, although there are no important reasons for this. Furthermore, the author [8] asserts in his article that the usually assigned to the group velocity physical sense of the rate of transfer of energy in this special case of the Tsennek's wave does not correspond to reality and therefore inequality (k) plays no role. Let us note that the chief conclusion of article [7] was reduced to the assertion, that rapid SW Tsennek's wave

does not possess the physical sense on the basis of the inequality (k). Therefore the author 8 declares in his article the article [7] of erroneous in this gist.

We see, thus, that discussion on SW Tsennek's wave it approached our culmination. Decoupling does depend on that, actually the rate of transfer of energy by this wave satisfies inequality (h) or this estimation is erroneous and in reality occurs reverse inequality (hh)  $V_E > c$ , which must be carried out, it is as soon as established accurately, that for this wave occurs the inequality (k) and in the weakly dissipative limit with the adequate accuracy it is carried out the usual equality:  $V_E = V_g$ .

## 6. Conclusion

The principles of relativistic mechanics are universal and required for any mechanical systems, including for the electromagnetic field, since it, as influential substance, possesses the mechanical properties, which are reflected in the density function of pulse and energy. In this connection a question about that, they can have these principles direct action in the theory of electromagnetism and as this generally must appear in the theory, it is not so already a negligible as the minimum. In the article is undertaken the attempt to solve this task and to verify this solution, where this especially urgent. Such region is, for example, the theory of surface electromagnetic waves, because the discussion about the physical sense of the rapid surface Tsennek's wave lasts here already more than hundred years and it is not still finished. It is shown that the application of principles of the relativistic mechanics in this region of the theory of electromagnetism proved to be the decisive factor: it is proven that the Tsennek's wave does not satisfy basic postulate SR. Corresponding to this wave solution of Maxwell's equations possesses that property, that the velocity of propagation of the density of its energy is more than the speed of light. Moreover, to correctly calculate the value of this speed beyond the limits of the methods of relativistic mechanics is impossible. It turns out that in this special case of the theory of electromagnetism relativistic mechanics played the positive role of censor, the thanks to which tightened discussion on the Tsennek's wave can be finished. If this is so, and if one considers that the principles of relativistic mechanics are universal, then, probably, does arise the more general task of checking that how does work this censorship in other sectors of the theory of electromagnetism?

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