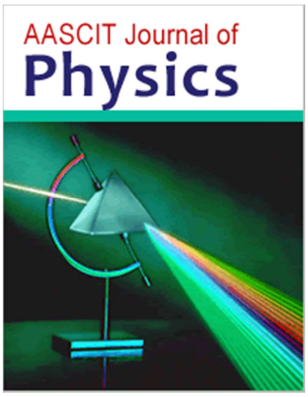
AASCIT Journal of Physics
2015; 1(4): 303-308
Published online July 20, 2015 (http://www.aascit.org/journal/physics)

American Association for Science and Technology



Keywords
Confinement,
a Quark,
Yang - Mills Field,
a Potential Barrier

Received: June 22, 2015
Revised: June 30, 2015
Accepted: July 1, 2015

The Possible Reason of Quarks Confinement in the Yang – Mills Field

Andrey N. Volobuev

Department of Physics, Samara State Medical University, Samara, Russia

Email address
volobuev47@yandex.ru

Citation
Andrey N. Volobuev. The Possible Reason of Quarks Confinement in the Yang – Mills Field. *AASCIT Journal of Physics.* Vol. 1, No. 4, 2015, pp. 303-308.

Abstract
On the basis of the analytical solution at the some approximation of the Yang - Mills field equations the proof of inevitable existence of the quarks confinement is received. It is shown, that as a result of components of the field nonlinear interaction on a way of a possible start of a quark from the hadron the configuration of the field as a potential barrier is formed. Physical consequences of inversely proportionality of the tensor-potential field components from the coupling constant are discussed.

1. Introduction

Confinement there is one of the most mysterious properties of quarks. It consists that the quark cannot leave the hadron. Moreover the quarks, probably, in general are not present in a free condition. They are absent, for example, in space rays. It is rather strange, since in the Universe there are processes to all possible energies. Especially exotic processes from the power point of view there are at the moment of the Universe occurrence. If the quark can be in a free condition it necessarily should be in space rays, at least as a relict of the Universe occurrence.

Absence of quarks in a free condition specifies two opportunities: or quarks are not present in the nature, or they cannot be in a free condition. The first opportunity is trivial and does not assume any proofs. The second opportunity assumes necessity of its proof.

Now as the experimental proof of quarks existence, first, results of deep inelastic scattering of leptons on nucleons, second, occurrence hadron's jets at annihilation electron – positron pairs of high energy of interaction [1] are considered.

Let's consider an opportunity of the quarks existence proof at presence of confinement.

The purpose of article is not research of various features of quarks, their conditions, characteristics, types, color, a spin, classifications, etc. The main attention is given to a power (gluon) field created by quarks.

The quark there is charged particle which creates around of itself so-called a gluon field. This field, probably, it is possible to describe the electromagnetic field similarly. But directly to apply the equations of an electromagnetic field to the quark gluon field it is impossible in connection with the big distinction in levels of power processes in the given fields. However undoubtedly in the description of the fields created by charged particles should be much in common.

2. The Yang – Mills Field Equations

Known extension of an electromagnetic field is Yang – Mills field [2].

Let's consider process of an electromagnetic field theory extension.

The principle of an electromagnetic field theory extension consists in use the gauge fields [3]. Technically it is do by replacement of a usual derivative on covariant derivative [4]:

$$\nabla_i = \frac{\partial}{\partial X_i} - ieL_i, \quad (1)$$

where L_i there is some the vector gauge field, e - in this case the electric charge which creates a field.

For example, using vector - potential of an electromagnetic field A we shall write down the strength of magnetic field as:

$$H = \text{rot}A = \begin{vmatrix} i & j & k \\ \nabla_X & \nabla_Y & \nabla_Z \\ A_X & A_Y & A_Z \end{vmatrix} \to \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial X} - ieL_X & \frac{\partial}{\partial Y} - ieL_Y & \frac{\partial}{\partial Z} - ieL_Z \\ A_X & A_Y & A_Z \end{vmatrix} =$$
$$= \text{rot}A - ie\begin{vmatrix} i & j & k \\ L_X & L_Y & L_Z \\ A_X & A_Y & A_Z \end{vmatrix} = \text{rot}A - ie[L \times A] = \text{rot}A + ie[A \times L] \quad (2)$$

If as gauge field to use a magnetic field $L = A$ the additional addend in (2) disappears, and we have usual expression of a magnetic field strength through its vector - potential $H = \text{rot}A$.

The extension of the electromagnetic field theory is convenient for carrying out in the 4-vector form. The tensor of the electromagnetic field strength in the 4-vector form looks like:

$$H_{\mu\nu} = \frac{\partial A_\nu}{\partial X_\mu} - \frac{\partial A_\mu}{\partial X_\nu} \quad (3)$$

where $\mu = 0,1,2,3$ and $\nu = 0,1,2,3$. The strength tensor $H_{\mu\nu}$ has $4 \times 4 = 16$ component which diagonal under $\mu = \nu$ obviously are equal to zero.

Let's extension the strength tensor of an electromagnetic field using a covariant derivative:

$$F_{\mu\nu} = \nabla_\mu B_\nu - \nabla_\nu B_\mu \quad (4)$$

where there is operator $\nabla_\mu = \frac{\partial}{\partial X_\mu} + igB_\mu$ .The value $F_{\mu\nu}$ is strength tensor of the Yang – Mills field which has $4 \times 4 = 16$ component, B_μ and B_ν - the tensor - potential of the Yang – Mills field, besides B_μ is also the tensor - potential of gauge field. All tensor have $4 \times 4 = 16$ components.

Passing in (4) to usual derivatives, we find:

$$F_{\mu\nu} = \frac{\partial B_\nu}{\partial X_\mu} - \frac{\partial B_\mu}{\partial X_\nu} + ig\left(B_\mu B_\nu - B_\nu B_\mu\right) = H_{\mu\nu} + igE_{\mu\nu} \quad (5)$$

where in this case $H_{\mu\nu} = \frac{\partial B_\nu}{\partial X_\mu} - \frac{\partial B_\mu}{\partial X_\nu}$ there is real part of the Yang – Mills field strength tensor which under our assumptions describes a gluon field of the quark, $E_{\mu\nu} = B_\mu B_\nu - B_\nu B_\mu = \begin{vmatrix} B_\mu & B_\nu \\ B_\mu & B_\nu \end{vmatrix}$ is an imaginary part of the field, g - the coupling constant (the strong interaction constant), specify the strong interaction. As the electric charge is a measure of the created electromagnetic field the coupling constant is a measure created by quarks gluon field. A constant in dimensionless form $\frac{g^2}{\hbar c} \approx 14$, where $\hbar$ there is reduced Planck constant, c - light speed in vacuum [5]. We shall notice that $B_\mu B_\nu - B_\nu B_\mu \neq 0$ since product of the Yang – Mills field tensor-potentials generally is non-commuting. The given circumstance essentially distinguishes the gluon field of a quark from a magnetic field (2).

Let's assume that the equations for a strength tensor of an electromagnetic field [6] in covariant form are applicable and for a strength tensor of the Yang – Mills field [7, 8]. Writing down these equations by means of covariant derivatives for a free field (without quark's currents) we have:

$$\nabla_\sigma F_{\mu\nu} + \nabla_\nu F_{\sigma\mu} + \nabla_\mu F_{\nu\sigma} = 0 \quad (6)$$

$$\nabla_\mu F_{\mu\nu} = 0 \quad (7)$$

In the equation (7) summation is carried out on repeating indexes.

Let's make some transformations to the equation (6). We shall pass to usual derivatives:

$$\begin{aligned}&\frac{\partial}{\partial X_\sigma}\left(\frac{\partial B_\nu}{\partial X_\mu}-\frac{\partial B_\mu}{\partial X_\nu}+ig\left(B_\mu B_\nu-B_\nu B_\mu\right)\right)+igB_\sigma\left(\frac{\partial B_\nu}{\partial X_\mu}-\frac{\partial B_\mu}{\partial X_\nu}+ig\left(B_\mu B_\nu-B_\nu B_\mu\right)\right)+\\&+\frac{\partial}{\partial X_\nu}\left(\frac{\partial B_\mu}{\partial X_\sigma}-\frac{\partial B_\sigma}{\partial X_\mu}+ig\left(B_\sigma B_\mu-B_\mu B_\sigma\right)\right)+igB_\nu\left(\frac{\partial B_\mu}{\partial X_\sigma}-\frac{\partial B_\sigma}{\partial X_\mu}+ig\left(B_\sigma B_\mu-B_\mu B_\sigma\right)\right)+\\&+\frac{\partial}{\partial X_\mu}\left(\frac{\partial B_\sigma}{\partial X_\nu}-\frac{\partial B_\nu}{\partial X_\sigma}+ig\left(B_\nu B_\sigma-B_\sigma B_\nu\right)\right)+igB_\mu\left(\frac{\partial B_\sigma}{\partial X_\nu}-\frac{\partial B_\nu}{\partial X_\sigma}+ig\left(B_\nu B_\sigma-B_\sigma B_\nu\right)\right)=0\end{aligned}\tag{8}$$

Resulting similar terms we shall find:

$$\begin{aligned}&\frac{\partial}{\partial X_\sigma}\left(B_\mu B_\nu-B_\nu B_\mu\right)+B_\sigma\left(\frac{\partial B_\nu}{\partial X_\mu}-\frac{\partial B_\mu}{\partial X_\nu}+ig\left(B_\mu B_\nu-B_\nu B_\mu\right)\right)+\\&+\frac{\partial}{\partial X_\nu}\left(B_\sigma B_\mu-B_\mu B_\sigma\right)+B_\nu\left(\frac{\partial B_\mu}{\partial X_\sigma}-\frac{\partial B_\sigma}{\partial X_\mu}+ig\left(B_\sigma B_\mu-B_\mu B_\sigma\right)\right)+\\&+\frac{\partial}{\partial X_\mu}\left(B_\nu B_\sigma-B_\sigma B_\nu\right)+B_\mu\left(\frac{\partial B_\sigma}{\partial X_\nu}-\frac{\partial B_\nu}{\partial X_\sigma}+ig\left(B_\nu B_\sigma-B_\sigma B_\nu\right)\right)=0\end{aligned}\tag{9}$$

Separately real and imaginary parts should be equal the equation (9) to zero. Equating (9) to zero the real part gives the equation:

$$\begin{aligned}&\frac{\partial}{\partial X_\sigma}\left(B_\mu B_\nu-B_\nu B_\mu\right)+B_\sigma\left(\frac{\partial B_\nu}{\partial X_\mu}-\frac{\partial B_\mu}{\partial X_\nu}\right)+\\&+\frac{\partial}{\partial X_\nu}\left(B_\sigma B_\mu-B_\mu B_\sigma\right)+B_\nu\left(\frac{\partial B_\mu}{\partial X_\sigma}-\frac{\partial B_\sigma}{\partial X_\mu}\right)+\\&+\frac{\partial}{\partial X_\mu}\left(B_\nu B_\sigma-B_\sigma B_\nu\right)+B_\mu\left(\frac{\partial B_\sigma}{\partial X_\nu}-\frac{\partial B_\nu}{\partial X_\sigma}\right)=0\end{aligned}\tag{10}$$

Differentiating products and resulting similar terms we receive:

$$\left(\frac{\partial B_\mu}{\partial X_\sigma}-\frac{\partial B_\sigma}{\partial X_\mu}\right)B_\nu+\left(\frac{\partial B_\sigma}{\partial X_\nu}-\frac{\partial B_\nu}{\partial X_\sigma}\right)B_\mu+\left(\frac{\partial B_\nu}{\partial X_\mu}-\frac{\partial B_\mu}{\partial X_\nu}\right)B_\sigma=0\tag{11}$$

The equation (11) is convenient for writing down with the help of the determinant (which top components are multiplied on the right):

$$\begin{vmatrix}B_\sigma & B_\nu & B_\mu\\ \frac{\partial}{\partial X_\sigma} & \frac{\partial}{\partial X_\nu} & \frac{\partial}{\partial X_\mu}\\ B_\sigma & B_\nu & B_\mu\end{vmatrix}=0\tag{12}$$

Equating to zero of the imaginary part of the equation (9) results in the following expression:

$$\begin{vmatrix}B_\sigma & B_\nu & B_\mu\\ B_\sigma & B_\nu & B_\mu\\ B_\sigma & B_\nu & B_\mu\end{vmatrix}=0\tag{13}$$

The feature of the equations (12) and (13) is absence in them the quark coupling constant g creating a field. Therefore these equations can have only auxiliary character. They are not connected to the mechanism of the field generation.

Let's consider the equation (7). Passing from covariant derivatives to usual derivatives we shall find:

$$\begin{aligned}&\frac{\partial}{\partial X_\mu}\left(\frac{\partial B_\nu}{\partial X_\mu}+igB_\mu B_\nu-\frac{\partial B_\mu}{\partial X_\nu}-igB_\nu B_\mu\right)+\\&+igB_\mu\left(\frac{\partial B_\nu}{\partial X_\mu}+igB_\mu B_\nu-\frac{\partial B_\mu}{\partial X_\nu}-igB_\nu B_\mu\right)=0\end{aligned}\tag{14}$$

Equating to zero the real part of the equation (14) we have:

$$\frac{\partial}{\partial X_\mu}\left(\frac{\partial B_\nu}{\partial X_\mu}-\frac{\partial B_\mu}{\partial X_\nu}\right)-g^2B_\mu\left(B_\mu B_\nu-B_\nu B_\mu\right)=0\tag{15}$$

Using above the accepted designations we shall write down:

$$\frac{\partial H_{\mu\nu}}{\partial X_\mu} - g^2 B_\mu E_{\mu\nu} = 0 \qquad (16)$$

The imaginary part of the equation (14) results in the equation:

$$\frac{\partial}{\partial X_\mu}\left(B_\mu B_\nu - B_\nu B_\mu\right) + B_\mu\left(\frac{\partial B_\nu}{\partial X_\mu} - \frac{\partial B_\mu}{\partial X_\nu}\right) = 0\,, \qquad (17)$$

or

$$\frac{\partial E_{\mu\nu}}{\partial X_\mu} + B_\mu H_{\mu\nu} = 0\,. \qquad (18)$$

The equation (16) is the basic equation reflecting occurrence of the field since the quark coupling constant g enters into it.

The equations (12), (13) and (16), (18) completely describe the Yang – Mills field created by a quark.

3. Solutions of the Yang – Mills Field Equations

Let's consider two kinds of possible solutions of the Yang – Mills field equations.

3.1. The Solution of Coulomb's Type

First of all we shall consider the special case of a free spherical-symmetric field of Coulomb's type. Such solution is well-known [3].

Let's assume all components of tensor-potentials of the field fall inverse proportional to the coordinate counted from the quark center $B_\mu = \frac{1}{X} b_\mu$, where is b_μ the matrix 4×4 with constant components. In this case the tensor $H_{\mu\nu} = -\frac{1}{X^2} h_{\mu\nu}$, where $h_{\mu\nu}$ there is matrix 4×4 having constant components. The tensor is $E_{\mu\nu} = \frac{1}{X^2} e_{\mu\nu}$, where $e_{\mu\nu}$ - also matrix 4×4 with constant components.

Hence, the basic equation (16) will be transformed to the kind:

$$2\sum_{\mu=0}^{3} h_{\mu\nu} = g^2 b_\mu e_{\mu\nu}\,. \qquad (19)$$

The equation (18) results in equality:

$$2\sum_{\mu=0}^{3} e_{\mu\nu} = b_\mu h_{\mu\nu}\,. \qquad (20)$$

The equation (12) or the equivalent equation:

$$H_{\sigma\mu} B_\nu + H_{\nu\sigma} B_\mu + H_{\mu\nu} B_\sigma = 0 \qquad (21)$$

transformed to the kind:

$$h_{\sigma\mu} b_\nu + h_{\nu\sigma} b_\mu + h_{\mu\nu} b_\sigma = 0\,. \qquad (22)$$

The equation (13) will be transformed to the kind:

$$\begin{vmatrix} b_\sigma & b_\nu & b_\mu \\ b_\sigma & b_\nu & b_\mu \\ b_\sigma & b_\nu & b_\mu \end{vmatrix} = 0\,. \qquad (23)$$

The equations (19), (20), (22), (23) allow, strictly speaking, to find all components ($4\times4\times4=64$) of the tensor-potentials $B_\mu = \frac{1}{X} b_\mu$ of the free spherical-symmetric quark field.

Thus, components of the tensor-potentials of a quark field fall inverse proportional to distance from a quark. This conclusion follows from the solution found actually by a trial and error method.

The received law of potential dependence from distance up to a quark creating a field is similar to corresponding law for an electric field. In the found dependence of tensor-potentials on distance obviously there is no indication on confinement existence.

3.2. The Solution of Confinement Type

However the equations of a quark field (12), (13), (16), (18) essentially differ from the Maxwell equations for an electromagnetic field first of all the nonlinearity which has cubic character. For the nonlinear equations there are various the solutions frequently essentially distinguished from each other reflecting various conditions of existing system. Therefore other solutions of the quark field equations can have specific features in comparison with spherical-symmetric solution found above.

For a finding of these features we shall consider the Yang – Mills under the following simplifying conditions:

$$\frac{\partial}{\partial X_0} = \frac{\partial}{\partial X_2} = \frac{\partial}{\partial X_3} = 0\,, \qquad (24)$$

where the condition $\frac{\partial}{\partial X_0} = 0$ reflects the stationary variant of the solved problem and changes of a field is only in the direction $X = X_1$.

Besides we shall believe:

$$B_2 = B_3 = 0\,, \qquad (25)$$

i.e. in directions X_2, X_3 the field is absent. Such conditions can be only modelling but not real. Conditions (25) are not obligatory for the analytical analysis of the problem but considerably reduce the calculation complexity.

Under conditions (24) and (25) equations (12) and (13), and hence the equation (6) are carried out identically. For example, at $\mu=0$ and $\nu=1$ determinants are

$$\begin{vmatrix} 0 & B_1 & B_0 \\ 0 & \dfrac{\partial}{\partial X_1} & 0 \\ 0 & B_1 & B_0 \end{vmatrix} = 0 \text{ and } \begin{vmatrix} 0 & B_1 & B_0 \\ 0 & B_1 & B_0 \\ 0 & B_1 & B_0 \end{vmatrix} = 0 .$$

In the equations (12), (13) and (18) there is no the coupling constant g therefore they describe only interaction between components of a quark field and cannot describe the confinement. For our purposes in modelling conditions (24) and (25) when half component $2\times4\times4$ is equal to zero the solution of these equations does not represent interest.

Let's address to the basic equation (16). Under conditions (24), (25) this equation looks like:

$$\frac{\partial H_{1\nu}}{\partial X_1} - g^2\left(B_0 E_{0\nu} + B_1 E_{1\nu}\right) = 0 . \quad (26)$$

For $\nu = 0$ taking into account $H_{10} = \dfrac{\partial B_0}{\partial X_1}$ and $E_{00} = 0$ we find:

$$\frac{\partial^2 B_0}{\partial X_1 \partial X_1} - g^2\left(B_1 B_1 B_0 - B_1 B_0 B_1\right) = 0 . \quad (27)$$

For $\nu = 1$ taking into account $H_{11} = 0$ and $E_{11} = 0$ we find:

$$B_0 B_0 B_1 = B_0 B_1 B_0 . \quad (28)$$

The most interesting is the equation (27) which we shall write down as:

$$\frac{\partial^2 B_0}{\partial X^2} = g^2\left(B_1 B_1 B_0 - B_1 B_0 B_1\right) . \quad (29)$$

Let's assume all components of the tensor-potentials B_0 and B_1 identically depend on coordinate X and they can be presented as $B_\mu = b_\mu f(X)$.

Hence the equation (31) will be transformed to the kind:

$$I\frac{\partial^2 f}{\partial X^2} = g^2 b_0^{-1}\alpha f^3 , \quad (30)$$

where $\alpha = b_1 b_1 b_0 - b_1 b_0 b_1$ there is a constant matrix 4×4, b_0^{-1} - a matrix which return to matrix b_0, I –unit matrix 4x4.

The solution of the equation (30) looks like [9]:

$$I f(X) = \sqrt{\frac{2}{b_0^{-1}\alpha}}\frac{1}{g(X - C)} , \quad (31)$$

where C there is any constant.

Thus the tensor-potentials depend under the law:

$$B_0 = b_0\sqrt{\frac{2}{b_0^{-1}\alpha}}\frac{1}{g(X - C)} , \; B_1 = b_1\sqrt{\frac{2}{b_0^{-1}\alpha}}\frac{1}{g(X - C)} . \quad (32)$$

Let's note also for the potentials of the kind (32) the equation (28) also is correct.

The tensor-potentials (32) have a line of features.

First of all they describe the phenomenon of confinement. At $X = C$ all components of tensor-potentials become equal to infinity.

On figure 1 the dependence of tensor-potentials components B_μ on distance up to the quark is shown in relative units.

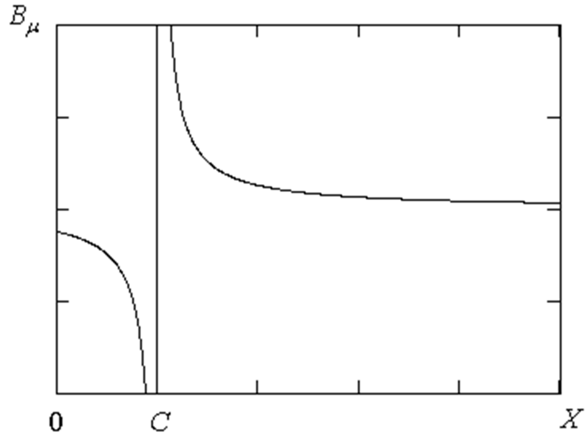


Figure 1. *The form of potential barrier creating the phenomenon of a quark confinement.*

Apparently from figure 1 the components of the tensor-potentials fall with increase in distance from a quark under the hyperbolic law. But on distance C there is impenetrable potential barrier. Therefore the quark cannot take off from a hadron.

The theory of the Yang - Mills field specifies only existence of a confinement but does not discover its physical reasons. Confinement there is consequence of the Yang - Mills field cubic nonlinearity i.e. its self-actions. As a result of the field components interaction on a way of a possible quark start from a hadron the field configuration as a potential barrier is formed. Here the analogy to other nonlinear process is correct. Before a plane flying with supersonic speed it is formed shock front or a surface of discontinuity which the plane cannot overcome. But the given analogy has essential difference from a quark model of hadron. The potential barrier in a hadron arises not only due to a quark which is taking off from hadron but also due to the quarks remaining in hadron. Unfortunately a finding of the Yang – Mills field equations (7) analytical solution at presence of the quark currents $\nabla_\mu F_{\mu\nu} = -J_\nu$ even in case of simplifications such as (24) and (25) is difficultly. Therefore the equations (6), (7) so-called a free field were used. As movement of quarks at the solution of the Yang – Mills field equations was not taken into account the potential barrier appeared motionless i.e. size $C = \text{const}$.

Till now we did not any serious assumptions except for the assumption that the field of quarks is Yang – Mills field.

The theory does not allow to find numerical value of size C. Apparently this size has the order of a nucleon diameter 10^{-15} m.

In connection with the carried out analysis there is a question: whether supervision of a free quark is possible? If the size C is proportional to speed of a taking off quark i.e. energy of influence on hadron in the interval $0-C$ the quark probably could be observed as free. If to estimate average energy of a quark in a nucleon 310 MeV (corresponding to 1/3 of a nucleon mass), and distance up to a potential barrier 10^{-15} m at the increase in energy of a quark, for example, up to 200 GeV the distance up to a potential barrier will increase up to $0.6 \cdot 10^{-12}$ m. This distance a quark flying with speed close by the speed of light in vacuum will fly for $0.2 \cdot 10^{-20}$ s. For energy influence on hadron 14 TeV which can be achieved in Large Hadron Collider the time of a quark life in a free condition $0.45 \cdot 10^{-18}$ s. I.e. time of a quark life in a free condition apparently is not enough it could be registered.

Other major difference of the received result (32) from the law for potential of an electric field is presence at tensor-potential of the coupling constant in the denominator. The potential of an electric field has an electric charge in numerator.

There is an interest the physical consequences of a coupling constant presence in a denominator to discuss. For this purpose we shall find out whether exist in the nature of force inversely proportional to a charge creating a field of these forces. The constant of thin structure of such forces should be inversely proportional to a square of the charge. For an electric charge such size should be proportional $\frac{\hbar c}{e^2} \approx 137$.

Strictly proved existence in the nature of such forces is not present however Dirac has hypothetically been predicted magnetic monopole satisfying this condition [5]. The given size for a magnetic monopole $\frac{137}{4}$. Existence of a magnetic monopole is necessary for the explanation of observable electric charge quantization. The common at a magnetic monopole and a quark is them not observability in the nature.

Therefore it is probable the physical nature of confinement is the common for a magnetic monopole and a quark. Especially in [10, 11] it is shown the magnetic monopole is topological not trivial solution in a class non-Abelian gauge fields to which the Yang – Mills field concerns also.

All assumptions concerning to the physical nature of the confinement are hypothetical. But existence of the confinement is strictly proved. It is consequence of cubic nonlinearity of the Yang – Mills field equations.

4. Conclusion

The quark confinement i.e. impossibility of its flying away from hadron and supervision in a free condition is consequence of nonlinearity of the gluon field equations which is presumably described by the Yang – Mills field theory. As a result of interaction of the field components on the way of a possible quark start from hadron the field configuration as a potential barrier is formed.

Feature of the quark field is presence in components of quark field tensor-potentials of the strong interaction constant in a denominator that allows assume the uniform physical nature of a quark and magnetic monopoles confinement.

References

[1] I.M. Dremin, A.B. Kaidalov, “Quantum chromodynamics and the phenomenology of strong interactions,” Moscow (2006), Uspekhi Fizicheskikh Nauk, Vol. 176, No. 3, pp. 275-287.

[2] N.N. Bogolubov, D.V. Shirkov, “Quantum Fields,” FIZMATLIT, Moscow (2005) p. 107.

[3] N.P. Konopleva, V.N. Popov, “Gauge fields,” Editorial URSS, Moscow (2000) pp. 38, 43.

[4] L.M. Zubov, M.I. Karjakin, “Tensor calculation,” High School Book, Moscow (2006) p. 95.

[5] “Physical encyclopedic dictionary,” Edit. A.M. Prokhorov, Sov. encyclopedia, Moscow (1983) pp. 309, 377.

[6] L.D. Landau, E.M. Lifshits, “Theory of Field,” Science, Moscow (1967) pp. 94, 104.

[7] I.M. Dremin, A.V. Leonidov, “The Quark-gluon Medium,” Moscow (2010), Uspekhi Fizicheskikh Nauk, Vol. 180, No. 11, pp. 1167-1196.

[8] F.J. Yndurain, “Quantum chromodynamics,” World, Moscow (1986) p. 234.

[9] E. Kamke, “Directory on Ordinary Differential Equations,” Science, Moscow (1976) p. 486.

[10] A.M. Polyakov, “Particle Spectrum in Quantum Field Theory,” JETP Lett., Vol. 20, No. 6, Moscow (1974) pp. 430-433.

[11] G. t'Hooft, “Magnetic monopoles in unified gauge theories,” Nuclear Physics. B. (1974) Vol. 79, pp. 276-284.