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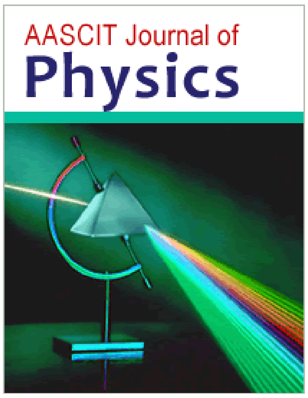

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Polytropic Stars with Tolman IV Type Potential

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Abstract
In this paper, we studied the behavior of relativistic objects with polytropic exponent for charged anisotropic matter distribution considering Tolman IV form for the gravitational potential Z. New exact solutions of the Einstein-Maxwell system are generated. The behavior of physical variables as energy density, charge density and radial pressure is consistent with seminal treatments which suggest relevance in the description of relativistic stars.

1. Introduction

From the development of Einstein´s theory of general relativity, the modelling of superdense mater configurations is an interesting research area (Kuhfitting, 2011; Bicak, 2006). In the last decades, such models allow explain the behavior of massive objects as neutron stars, quasars, pulsars, black holes and white dwarfs (Malaver,2013a; Komathiraj&Maharaj,2008; Sharma *et. al*.,2001).

In theoretical works of realistic stellar models, is important include the pressure anisotropy (Bowers & Liang, 1974; Cosenza et al., 1981; Gokhroo&Mehra, 1994). Bowers & Liang (1974) extensively discuss the effect of pressure anisotropy in general relativity. The existence of anisotropy within a star can be explained by the presence of a solid core, phase transitions, a type III super fluid, a pion condensation (Sokolov, 1980) or another physical phenomena as the presence of an electrical field (Usov, 2004).The physics of ultrahigh densities is not well understood and many of the strange stars studies have been performed within the framework of the MIT bag model (Komathiraj & Maharaj, 2007). In this model, the strange matter equation of state has a simple linear form given by $p = \frac{1}{3}(\rho - 4B)$ where ρ is the energy density, p is the isotropic pressure and B is the bag constant. Many researchers have used a great variety of mathematical techniques to try to obtain exact solutions for quark stars within the framework of MIT bag model, since it has been demonstrated by Komathiraj and Maharaj (2007), Malaver (2009,2014b), Thirukkanesh and Maharaj (2008), Maharaj *et al*. (2014), Thirukkanesh and Ragel (2013) and Sunzu *et al*. (2014). With then use of Einstein´s field equations, important advances has been made to model the interior of a star. In particular, Feroze & Siddiqui (2011) and Malaver (2014a, 2015) consider a quadratic equation of state for the matter distribution and specify particular forms for the gravitational potential and electric field intensity. MafaTakisa and Maharaj (2013) obtained new exact solutions to the Einstein-Maxwell system of equations with a polytropic equation of state. Thirukkanesh and Ragel (2012) have obtained particular models of anisotropic fluids with polytropic equation of state which are consistent with the reported experimental observations. Malaver (2013b,2013c) generated new exact solutions to the Einstein-Maxwell system considering Van der Waals modified equation of state with and without polytropical exponent and Thirukkanesh and Ragel (2014) presented a anisotropic strange quark matter model by imposing a linear barotropic equation of state with Tolman IV form for the gravitational potential. Mak and Harko (2004) found a relativistic model of strange quark star with the suppositions of spherical symmetry and conformal Killing vector.The objective of this paper is to obtain new exact solutions to the Maxwell-Einstein system for anisotropic matter with a polytropic equation of state in presence of an electromagnetic field using Tolman IV form for the gravitational potential Z. We have obtained some new classes of static spherically symmetrical models for particular polytropic indices. This article is organized as follows, in Section 2, we present Einstein´s field equations of anisotropic fluid distribution. In Section 3, we make a particular choice of gravitational potential Z(x) that allows solving the field equations and we have obtained new models for charged anisotropic matter. In Section 4, a physical analysis of the new solutions is performed. Finally in Section 5, we conclude.

2. Einstein Field Equations

We consider a spherically symmetric, static and homogeneous and anisotropic spacetime in Schwarzschild coordinates given by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (1)$$

where $\nu(r)$ and $\lambda(r)$ are two arbitrary functions.

The Einstein field equations for the charged anisotropic matter are given by

$$T_{00} = -\rho - \frac{1}{2}E^2 \quad (2)$$

$$T_{11} = p_r - \frac{1}{2}E^2 \quad (3)$$

$$T_{22} = T_{33} = p_t + \frac{1}{2}E^2 \quad (4)$$

where ρ is the energy density, p_r is the radial pressure, E is electric field intensity and p_t is the tangential pressure, respectively. Using the transformations, $x = cr^2$, $Z(x) = e^{-2\lambda(r)}$ and $A^2y^2(x) = e^{2\nu(r)}$ with arbitrary constants A and c>0, suggested by Durgapal and Bannerji (1983), the metric (1) take the form
and the Einstein field equations can be written as

$$ds^2 = -A^2y^2(x)dt^2 + \frac{1}{4cxz}dx^2 + \frac{x}{c}(d\theta^2 + \sin^2\theta d\varphi^2) \quad (5)$$

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{c} + \frac{E^2}{2c} \quad (6)$$

$$4Z\frac{\dot{y}}{y} - \frac{1-Z}{x} = \frac{p_r}{c} - \frac{E^2}{2c} \quad (7)$$

$$4xZ\frac{\ddot{y}}{y} + (4Z + 2x\dot{Z})\frac{\dot{y}}{y} + \dot{Z} = \frac{p_t}{c} + \frac{E^2}{2c} \quad (8)$$

$$\sigma^2 = \frac{4cZ}{x}\left(x\dot{E} + E\right)^2 \quad (9)$$

σ is the charge density and dots denote differentiation with respect to x.

Following Thirukkanesh and Ragel (2012), we assume a polytropic equation of state relating the radial pressure to the energy density given by

$$p_r = \alpha\rho^\Gamma \quad (10)$$

where $\Gamma = 1 + 1/\eta$, η is the polytropic index and α is arbitrary constant.

3. The New Models

In order to solve Einstein-Maxwell's field equations, we have chosen specific forms for the gravitational potential Z and the electrical field intensity E. Motivated by Tolman (1939) and Feroze and Siddiqui (2014), we make the specific choices

$$Z(x) = \frac{(1+ax)(1-bx)}{(1+2ax)} \quad (11)$$

$$E^2 = \frac{2c(1-Z)}{x} \quad (12)$$

The potential is regular at the origin and well behaved in the interior of the sphere. Substituting (11) in (12), for the electric field we have

$$E^2 = \frac{2c(a+b+abx)}{1+2ax} \quad (13)$$

E is finite at the centre of star and remains continuous in the interior. With eq.(11), (12) and (13), we have found the following expressions for ρ, p_t, σ^2 and the metric function $e^{2\lambda}$

$$\rho = 2c\frac{(a+b+2abx+2a^2bx^2)}{(1+2ax)^2} \quad (14)$$

$$p_t = \frac{4xc(1+ax)(1-bx)}{(1+2ax)}\frac{\ddot{y}}{y} + 2c\left[\frac{4+2(5a-3b)x+8a(a-2b)x^2-12a^2bx^3}{(1+2ax)^2}\right]\frac{\dot{y}}{y} - c\frac{\left[2a+2b+a(2a+5b)x+4a^2bx^2\right]}{(1+2ax)^2} \quad (15)$$

$$\sigma^2 = \frac{2c^2(1+ax)(1-bx)\left[4a^2bx^2+a(2a+5b)x+2(a+b)\right]^2}{x(1+2ax)^4(a+b+abx)} \quad (16)$$

$$e^{2\lambda} = \frac{(1+2ax)}{(1+ax)(1-bx)} \quad (17)$$

In this paper, we consider three cases of polytropic index $\eta=\infty$, 1/2, 1/3 when anisotropy and the electromagnetic field are present.

For the case $\eta\to\infty$, substituting (14) in eq.(10), the radial pressure can be written in the form

$$p_r = 2\alpha c\frac{\left(a+b+2abx+2a^2bx^2\right)}{(1+2ax)^2} \quad (18)$$

Substituting (18), (13) and (11) in (7), we have

$$\frac{\dot{y}}{y} = \frac{\alpha\left(a+b+2abx+2a^2bx^2\right)}{2c(1+2ax)(1+ax)(1-bx)} \quad (19)$$

Integrating (19), we obtain

$$y(x) = c_1(1+ax)^A(-1+bx)^B(1+2ax)^C \quad (20)$$

where

$$A = B = -\frac{\alpha}{2c}, \text{ and } C = \frac{\alpha}{2c} \quad (21)$$

The metric function $e^{2\nu}$ can be written as

$$e^{2\nu} = A^2c_1^2(1+ax)^{2A}(-1+bx)^{2B}(1+2ax)^{2C} \quad (22)$$

The metric for this model is

$$ds^2 = -A^2c_1^2(1+ax)^{2A}(-1+bx)^{2B}(1+2ax)^{2C}dt^2 + \frac{(1+2ax)}{4xc(1+ax)(1-bx)}dx^2 + \frac{x}{c}(d\theta^2+\sin^2\theta d\varphi^2) \quad (23)$$

With $\eta=1/2$ and replacing (14) in (10), we have for the radial pressure

$$p_r = 8\alpha c^3\frac{\left(a+b+2abx+2a^2bx^2\right)^3}{(1+2ax)^6} \quad (24)$$

Substituting (11),(13) and (24) in (7), we have

$$\frac{\dot{y}}{y} = \frac{2\alpha c^2\left(a+b+2abx+2a^2bx^2\right)^3}{(1+2ax)^5(1+ax)(1-bx)} \quad (25)$$

Integrating (25), we obtain

$$y(x) = c_2(1+2ax)^D(1+ax)^E(-1+bx)^F\exp[G(x)] \quad (26)$$

The values of the constants D, E, F and the variable G(x) appear in the appendix $e^{2\nu}$ can be written as

$$e^{2\nu} = A^2c_2^2(1+2ax)^{2D}(1+ax)^{2E}(-1+bx)^{2F}\exp[2G(x)] \quad (27)$$

The metric for this model is

$$ds^2 = -A^2c_2^2(1+2ax)^{2D}(1+ax)^{2E}(-1+bx)^{2F}\exp[2G(x)]dt^2 + \frac{(1+2ax)}{4xc(1+ax)(1-bx)}dx^2 + \frac{x}{c}(d\theta^2+\sin^2\theta d\varphi^2) \quad (28)$$

With $\eta=1/3$, the expressions for p_r, $y(x)$ and $e^{2\nu}$ are given for

$$p_r = 16\alpha c^4\frac{\left(a+b+2abx+2a^2bx^2\right)^4}{(1+2ax)^8} \quad (29)$$

$$y(x) = c_3(1+2ax)^H(1+ax)^I(-1+bx)^J\exp[K(x)] \quad (30)$$

$$e^{2\nu} = A^2c_3^2(1+2ax)^{2H}(1+ax)^{2I}(-1+bx)^{2J}\exp[2K(x)] \quad (31)$$

Again the values of the constants H, I, J and the variable K(x) appear in the appendix

The metric for this model is

$$ds^2 = -A^2c_3^2\left(1+2ax\right)^{2H}\left(1+ax\right)^{2I}\left(-1+bx\right)^{2J}\exp\left[2K(x)\right]dt^2 + \frac{\left(1+2ax\right)}{4xc\left(1+ax\right)\left(1-bx\right)}dx^2 + \frac{x}{c}(d\theta^2 + \sin^2\theta d\varphi^2) \quad (32)$$

Figures 1, 2, 3, 4, 5 and 6 represent the graphs of p_r and radial speed of sound v_{sr}^2 with $\eta = \infty$, 1/2 and 1/3, respectively. The graphs has been plotted for a particular choice of parameters a = 0.0169, b = 0.00454, α=1/3 with a stellar radius of r=10 Km.

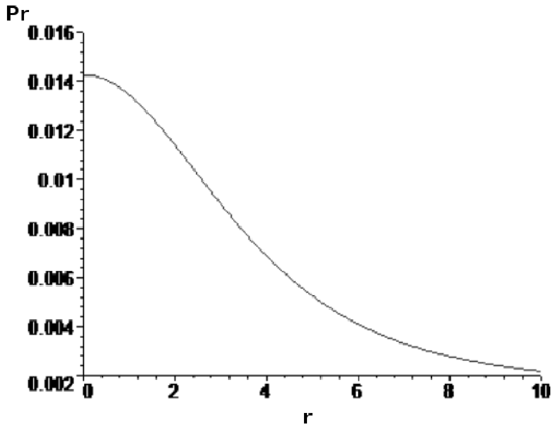

Fig. 1. *Radial Pressure with η →∞.*

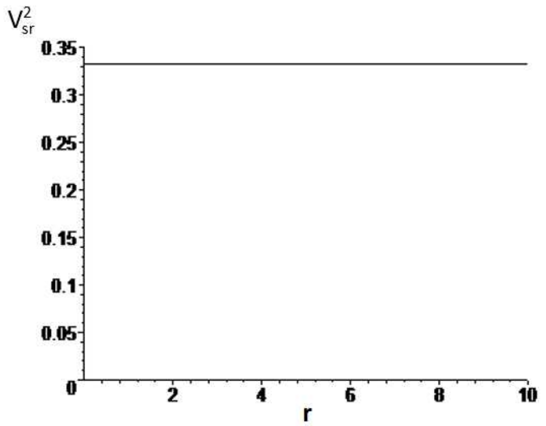

Fig. 2. *Radial speed of sound with η →∞.*

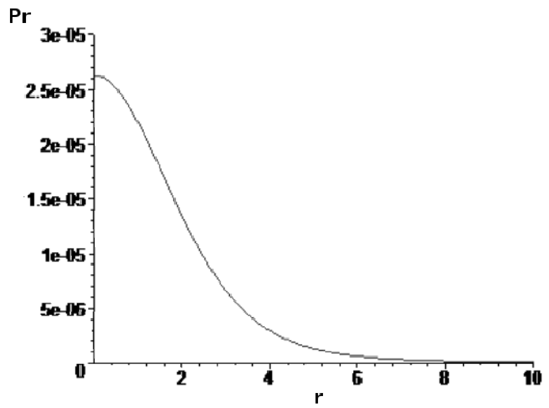

Fig. 3. *Radial pressure with η=1/2.*

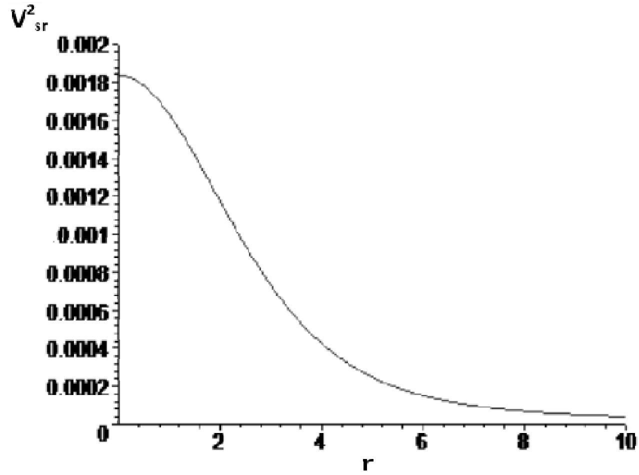

Fig. 4. *Radial speed of sound with η=1/2.*

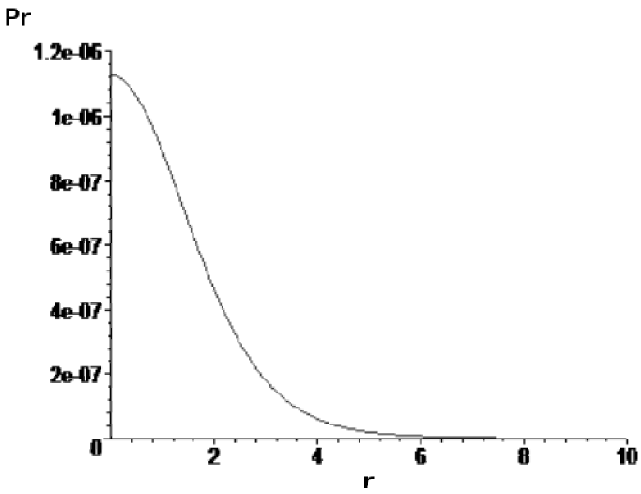

Fig. 5. *Radial Pressure with η =1/3.*

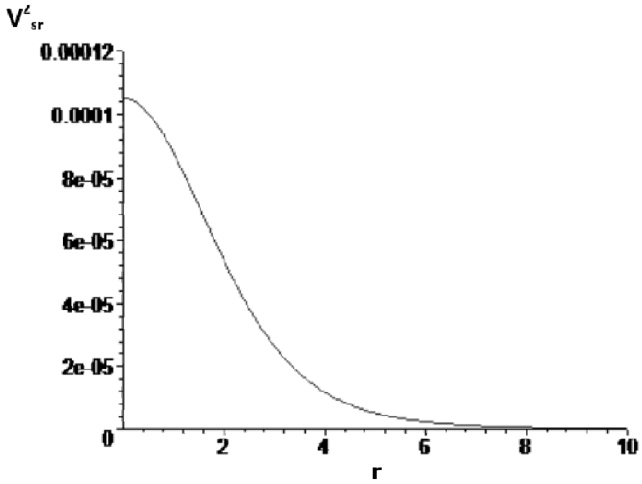

Fig. 6. *Radial speed of sound with η=1/3.*

4. Physical Features of the New Models

Any physically acceptable solutions must satisfy the following conditions:

(i) Regularity of the gravitational potentials in the origin.
(ii) Radial pressure must be finite at the centre.
(iii) $P_r > 0$ and $\rho > 0$ in the origin.

With $\eta \to \infty$, the gravitational potentials are regular at the origin since $e^{2\nu(0)} = A^2 c_1^2 (-1)^{2B}$ and $e^{2\lambda(0)} = 1$ are constants and $\left(e^{2\lambda(r)}\right)' = \left(e^{2\nu(r)}\right)' = 0$ at r=0. In the centre $\rho(0) = 2c(a+b)$ and $P_r = \frac{2}{3}c(a+b)$ both are positive if a > 0 and b > 0.

For the case $\eta = 1/2$ $e^{2\lambda(0)} = 1$,

$$e^{2\nu(0)} = A^2 c_2^2 (-1)^{2F} e^{\frac{c^2(56a^3+116a^2b+14ab^2-27b^3)}{72(a+b)}}$$

in the origin $r=0$ and $\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0$. This shows that the potential gravitational is regular in the origin. In the centre $P_r = \frac{8}{3}c^3(a+b)^3$ and $\rho(0) = 2c(a+b)$.

With $\eta = 1/3$, $e^{2\lambda(r)} = 1$,

$$e^{2\nu(0)} = A^2 c_3^2 (-1)^{2J} e^{\frac{c^3(1184a^5-415b^5+4448a^4b+2732a^2b^3+6056a^3b^2-376ab^4)}{360(2a+b)^2}}$$

in the origin and. $\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0$. Again the gravitational potential is regular in $r=0$. The energy density is $\rho = 2c(a+b)$ and the radial pressure $P_r = \frac{16}{3}c^4(a+b)^4$ at r=0. In all the cases the charge density is continues inside of the star and singular in the center.

In fig.1,3 and 5 the radial pressure is finite and decreasing for all studied cases. Observe that the profiles of the radial pressure diminishes as a function if the radial coordinate when decreases the polytropic index. For the case $\eta \to \infty$, the isothermal sphere is generated. To maintain of causality, the square of sound speed defined as $v_{sr}^2 = \frac{dp_r}{d\rho}$ should be within the limit $0 \le v_{sr}^2 \le 1$ in the interior of the star. In fig 2, 4 and 6, this condition is maintained in all the studied models.

5. Conclusion

Inthis paper, we have generated new exact solutions to the Einstein-Maxwell system with a polytropic equation of state and considering Tolman IV form for the gravitational potential Z. The new obtained models may be used to model relativistic stars in different astrophysical scenes. The relativistic solutions to the Einstein-Maxwell systems presented are physically reasonable. The charge density σ is singular at the origin and behaves well in the stellar interior. The gravitational potentials are regular at the centre and well behaved.

When the index $\eta \to \infty$ it recovered the bag equation of state $p_r = \alpha\rho$ that correspond to an isothermal sphere. The models presented in this article may be useful in the description of relativistic compact objects with charge, strange quark stars and configurations with anisotropic matter.

Appendix

On integrating the equation (25), the values of the constants D, E, F and the variable G (x) are given by

$$D = \frac{\alpha c^2\left(52a^2b^2 + 28ab^3 + 7b^4 + 16a^4 + 48a^3b\right)}{2(b+2a)^2} \tag{A.1}$$

$$E = -2\alpha c^2\left(a^2 + 2ab + b^2\right) \tag{A.2}$$

$$F = \frac{-2\alpha b^2 c^2\left(a^2 + 2ab + b^2\right)}{(b+2a)^2} \tag{A.3}$$

$$G(x) = \frac{\alpha c^2\begin{pmatrix} 768a^6x^3 + 1920a^5bx^3 + 960a^5x^2 + 2400a^4bx^2 + 960a^4b^2x^3 + 448a^4x + 56a^3 \\ +1056a^3b^2x^2 + 1024a^3bx - 96a^2b^3x^2 + 116a^2b + 352a^2b^2x - 96ab^3x \\ +14ab^2 - 27b^3 \end{pmatrix}}{24(b+2a)(1+2ax)^4} \tag{A.4}$$

For the equation (30), the values of H, I, J and the variable K(x) take the form

$$H = \frac{\alpha c^3\left(64a^6 + 288a^5b + 528a^4b^2 + 504a^3b^3 + 276a^2b^4 + 90ab^5 + 15b^6\right)}{2(b+2a)^3} \tag{A.5}$$

$$I = -4\alpha c^3\left(a^3 + 3a^2b + 3ab^2 + b^3\right) \tag{A.6}$$

$$J = -\frac{4\alpha b^3 c^3 \left(a^3 + 3a^2 b + 3ab^2 + b^3\right)}{\left(b + 2a\right)^3} \tag{A.7}$$

$$K(x) = \frac{\alpha c^3 \begin{pmatrix} 576000a^6b^3x^4 + 55296a^5bx - 8220a^2b^5x^2 + 65280a^6b^4x^5 + 361440a^5b^2x^2 \\ 80352a^4b^2x + 221040a^4b^3x^2 + 34560a^3b^4x^2 + 384000a^8b^2x^5 + 245760a^9bx^5 \\ +792320a^6b^2x^3 + 510720a^5b^3x^3 + 44784a^3b^3x + 512000a^7bx^3 + 101760a^4b^4x^3 \\ +552960a^8bx^4 - 10560a^3b^5x^3 - 5280a^4b^5x^4 + 864000a^7b^2x^4 + 238080a^6bx^2 \\ +261120a^7b^3x^5 + 131520a^5b^4x^4 - 2940ab^5x + 3648a^2b^4x + 1184a^5 - 415b^5 \\ +4448a^4b + 2732a^2b^3 + 6056a^3b^2 + 128000a^8x^3 + 59520a^7x^2 + 14208a^6x \\ -376ab^4 + 61440a^{10}x^5 + 138240a^9x^4 \end{pmatrix}}{120\left(1 + 2ax\right)^6 \left(2a + b\right)^2} \tag{A.8}$$

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