10

Brief report on electromagnetic invisibility

10.1 Concept of electromagnetic invisibility

Invisibility has been a long-lasting dream of scientists, writers, artists, movie makers, and many more. The notion of invisibility carries a magical aura that sparkles our dreams and make us believe that everything else becomes possible. Certainly achieving invisibility would open the door to a world of applications limited only by our imaginations. The possibility of invisibility, however, has belonged to the realm of mythological tales and science fiction rather than to our real world, until the 21st century. While the invisibility in the tales of yore stemmed purely from "magic" with no attempts to relating the phenomena to actual physics, science fiction has often tried to explore such plausible links. The "invisible man" in Wells (2002) for example used a chemical drink that turned the refractive index of his body identical to that of air thus making him transparent. However, the scientific analogy remained relatively shallow, neglecting the fact that the person would be completely blind if perfectly index matched – the very process of sight crucially depending on absorption of light in the retina, as pointed out in Perelman (1913).

In the scientific quest, two main approaches to achieve invisibility have been taken, active and passive. The active approach is more straightforward and consists of taking real-time pictures of a scene and displaying them on a screen that is actually hiding or blocking the scene itself. For example, the screen can be the coat worn by a person, onto which is displayed in real time the scene behind the person, effectively giving the illusion of transparency to an observer from the front. This virtual reality approach has been reported in (Japanese scientist invents 'invisibility coat'), for example. It has also been reported that the armed forces across the world have toyed with some form or other of active camouflage using virtual reality. However, the actual field use of such virtual reality techniques is not known as yet.

The passive approach, on the contrary, relies on the intrinsic properties of the object or subject to not scatter the incident wave in an usual way and to send confusing information to the receiver. In this regard, two subtypes of invisibility can be distinguished: one where the object is merely not identifiable and the other one where the object does not affect the radiation field (except at very nearby regions of space) so that it cannot be sensed

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by an observer some distance away and can be considered to be actually invisible. In electromagnetic or optical language, rendering an object not identifiable means to significantly lower its scattering cross-section, with the ideal situation of lowering it down to zero, at which point the object does not scatter anymore the incident fields. Such reduction can for example be achieved using good impedance matching and very absorbing materials: the material (ideally) totally absorbs the incident power and nothing is reflected back to the receiver, which therefore cannot identify the object in front of it.* This is the approach that has been traditionally undertaken for military applications in order to render objects unidentifiable on radar screens. Note that merely having a highly absorbing material does not suffice, as such media are usually poorly impedance matched to vacuum in which case the scattered energy dominates over the absorbed energy. A simple manner of impedance matching for a highly absorbing medium would be to have large imaginary parts (compared to the real parts) for both the permittivity and permeability of the medium so that $(\mu/\varepsilon)^{1/2} \sim [\mathrm{Im}(\mu)/\mathrm{Im}(\varepsilon)] \sim 1$. Ideally one could use anisotropic media to realize impedance matching via perfectly matched layer boundary conditions (Berenger 1994) so that all incident radiation is guided into the medium where it is absorbed.

In the second case of actual invisibility, the object can either allow the light to be seen through (which is, for example, the purpose of the virtual reality process mentioned above or in the sense of the invisible man) or smoothly guide the light around itself like a fluid flowing in streamlines around an obstacle. The second option would be possible by placing around the object another cloaking material that guides the light around the object through the cloak which would have to be a “see through” material. These approaches result in not affecting the radiation at any point some distance away from the object, which is the ultimate goal of invisibility. Note that in the first case in which the object would allow the light to go through itself, the processes of invisibility would crucially depend on the properties of the object. In the second option interestingly, the design of the invisibility cloak becomes independent of the properties of the object itself.

There is, however, a fundamental difference in the two passive invisibilities highlighted above. In the situation of a zero scattering cross-section, the extinction coefficient is still non-zero, i.e. the object is not visible but would still block the rays if it were located between a source and a receiver. The receiver therefore would not know what type of object is in front of it, but it would still know that some perturbing element is present whose location and other properties can be estimated, and perhaps deduced from the shadow. Hence, even

*We refer here to active observation, most commonly, where a wave is first sent onto an object, reflects from it, and reaches a receiver. This is, for example, the manner of our vision: light constantly bounces off the objects surrounding us, reaching our eye. In the case of luminous objects, the receiver passively observes radiation emitted directly by the object.

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the sole knowledge of the existence of the object is important information. On the other hand, if the object is seen through or excluded, the existence itself of the object disappears since the environment is indistinguishable whether there is or there is no object. Therefore, it is not a situation where the observer receives no information, but instead it is a situation where the observer receives wrong information which leads him to an erroneous conclusion (such as "the space is empty of scatterers") and does not spark any suspicion.

Remarkably, metamaterials have been very instrumental at enabling invisibility and cloaking from electromagnetic waves, primarily due to their ability to make anisotropic metamaterials with the required material parameters.

All the previous chapters have been devoted to the presentation of some aspects of metamaterials: their genesis, their physical implementation, as well as their major properties as they have been investigated and reported in the open literature for about a decade. Many of these properties had already been mentioned in what is considered the first paper directly related to this topic Veselago (1968), although without so much in-depth analysis. These properties revolved around the concepts of negative permittivity ε and permeability μ, negative refraction, and their consequences on otherwise well-known electromagnetic phenomena: refraction, Doppler shift, Čerenkov radiation, imaging, etc.

Over the last decade, the major step forward from Veselago (1968) has been the experimental reality of these media in the form of metamaterials. After the fundamental papers providing ideas on how to design metamaterials to achieve $\varepsilon < 0$ and $\mu < 0$ (Pendry et al. 1998; 1999), various communities of researchers made intense efforts to elaborate, optimize, and further develop upon these first concepts. As a result, it is now possible to tailor the material properties to a level unachieved before, accessing all quadrants of the $(\mathrm{Re}\{\varepsilon\}, \mathrm{Re}\{\mu\})$ plane shown in Fig. 1.10. In addition to only controlling the sign of the constitutive parameters, scientists have learned how to induce and tailor anisotropic properties and even bianisotropic properties to some extent. From this perspective, the unprecedented control of material properties offered by metamaterials is at its infancy. The latter have been discovered by trying to achieve negative constitutive parameters, but have opened a door much wider than the original intended purpose. The quest for invisibility can be viewed as a second domain of application of metamaterials, which may yield as unexpected and revolutionary ideas as those prompted by the quest for negative constitutive parameters,

The concept of invisibility using metamaterials has appeared in the literature very recently, and is currently mainly based on two independent approaches, already highlighted in the paragraphs above. The first, proposed in Alù and Engheta (2005), takes advantage of the negative polarizability of left-handed media in order to compensate for the polarizability of regular dielectrics and to lower their scattering cross-section. This, however, requires a detailed knowledge of the electromagnetic scattering properties of the object

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in order to design the compensating cloak for it. A more detailed discussion on this technique is provided in Section 10.3.

The second approach proposed in Pendry et al. (2006) to achieve invisibility relies on the proper control of the constitutive parameters within a certain region of space such as to smoothly deviate and guide the incident light around a core region, and to return the rays to their original propagation path upon leaving that same region. In this approach, the cloaked object placed in the core region never interacts with the radiation and the design of the cloak becomes independent of the cloaked object. This approach, where invisibility arises from exclusion from the electromagnetic fields, is discussed subsequently. An analogous approach but based mainly on ray analysis was independently developed by Leonhardt (2006a;b).

10.2 Excluding electromagnetic fields

10.2.1 Principle

The concept of electromagnetic invisibility is fairly simple to grasp and is represented in Fig. 10.1: a region of space is invisible to an observer if it does not affect the light going through it or in its vicinity differently than the medium in which it is embedded. In fact, it has been claimed that true invisibility is impossible on the basis of the uniqueness of the inverse scattering problem: a full knowledge of the phase and amplitudes of the scattered fields in all directions enables a unique specification of the scatterer's spatial distribution of the refractive index (Nachman 1988, Wolf and Habashy 1993). However, this neglects that both magnetic permeability and the dielectric permittivity can be anisotropic, independently specified, and that scattering systems with different local resonant fields can have similar far-field scattering properties.

In common words, this concept of invisibility is described as "see through." It is important to realize that the observer has no knowledge of what happens inside the invisible region of space, insofar as it does not introduce detectable perturbations of the surrounding rays. In Fig. 10.1 for example, the connection between points A_1 and A_2 can be arbitrary, as long as the phase, the amplitude, and the direction of the waves are leaving the region as they would have been if there had been no object. The challenge to achieve invisibility is therefore to use the phenomenon of refraction in order to design a region of space with the right properties so that the rays that enter it seemingly leave it without any perturbation.

It is often the case that natural phenomena provide ideas to physicists and engineers to find solutions to new problems. The present situation is an additional example of such case, and an analogy with a totally unrelated field

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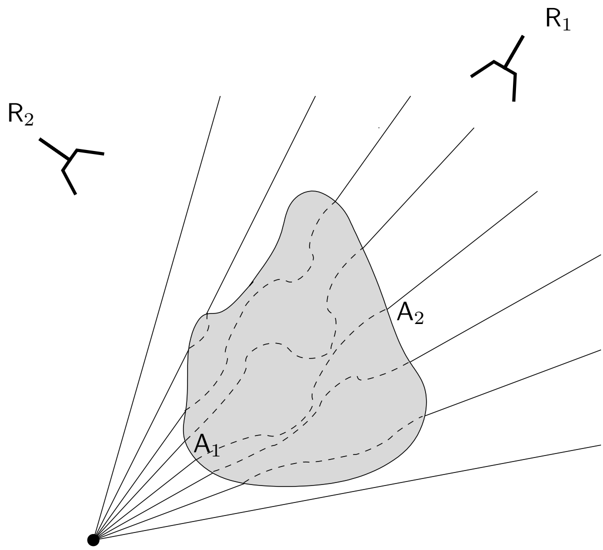

Figure 10.1 Illustration of the concept of invisibility: the gray region of space is invisible to both receivers R_1 and R_2 if it does not perturb the rays outside it. The receivers, however, do not need to know the ray distribution inside the region and the path between A_1 and A_2 can be arbitrary as far as the phase, amplitude, and directions are preserved from the two points.

can be provided, *viz.*, the theory of General Relativity.

General Relativity is a cosmological theory of space and time established by Einstein (Einstein 1916) that explains the trajectory of planets and other celestial bodies as a deformation of the space-time continuum due to mass and energy. Hence, unlike the Newtonian theory of gravitation which involves a force that perturbs the motion of planets from their straight-line trajectories in the Euclidian space, Einstein's theory shows that all trajectories are in fact geodesics but in a Riemann curved space-time. Consequently, the trajectories of planets as we observe them is merely the projection of such geodesic from the real four-dimensional (4D) space (three dimensions of space and one of time) onto our more common three-dimensional (3D) space. In other words, in our 3D Euclidian space, the geodesics, which can be viewed as straight lines in the 4D space, appear curved.

There is a simple leap from these ideas to the concept of invisibility. Suppose that what is achieved by mass and energy in the general theory of relativity can be achieved by altering the material properties of a medium; then we should be able to bend the rays along predefined geodesics inside the medium and return them to straight paths (which are also geodesics in a different space or a different medium in our case) at will. This concept is illustrated in Fig. 10.2: a circular region of space is created with such constitutive parameters to

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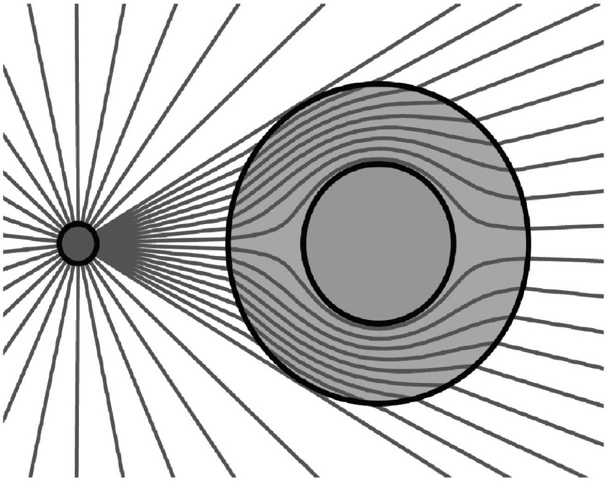

Figure 10.2 Bending the rays along geodesics: the path of rays is modified by the proper material parameters and returned to straight lines upon leaving the medium, thus achieving invisibility. A proper design of geodesics can also cloak an object in the center of the structure. (Reproduced from Pendry et al. (2006). Reprinted with permission from AAAS.)

bend the rays as shown. Again, the rays are bent in this way because we observe them from an Euclidian space, whereas they correspond to geodesics inside the medium. The rays therefore travel along straight paths in free-space, "staight" paths inside the medium (although they appear curved to us observing from free-space), and are returned to regular straight paths upon leaving the medium and re-entering free-space. Comparing the field outside the region with the illustration of Fig. 10.1, we see a striking similarity and the direct connection to the concept of invisibility. This is the approach based on ray analysis developed in Leonhardt (2006a;b) for inhomogeneous but isotropic refractive media.

Notice in addition a striking feature of the configuration in Fig. 10.2: the central region of the medium is totally shielded from any radiation. An object of arbitrary shape can be located in this region and would not perturb the radiation in the surrounding regions, which would then propagate as if the object were not there. The object is therefore cloaked with an invisibility shield and cannot be detected.† By reciprocity, any fields emitted by the object inside the shielded volume would be confined to that volume only, without any possibility of propagating out of the cloaked volume. This design requires anisotropic materials and independent ε and μ for its function.

†As we shall see subsequently, such cloaking only works perfectly at a single frequency.

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10.2.2 Design procedure

The constitutive parameters that achieve a ray diagram of the type shown in Fig. 10.2 are very specific and need to be properly computed. They result from transforming the fields in one geometry (typically the regular 3D free-space) onto another (a sphere in Fig. 10.2) or vice versa, and therefore are obtained by a properly chosen change of coordinates (or mapping). This is a very similar approach to the one already taken in Section 9.3 to realize a perfect lens with geometries other than an infinite slab. The main transformation relation was shown to be Eq. (9.31b), which yields the constitutive parameters $\bar{\bar{\varepsilon}}(\mathbf{r})$ and $\bar{\bar{\mu}}(\mathbf{r})$ in the new coordinate system. As already suggested by the notation and explained in Section 9.3.1, the constitutive parameters become anisotropic and spatially varying. The Maxwell equations (9.30) with the generalized constitutive parameters in place are, however, form invariant so that all the necessary electromagnetic results in homogeneous, isotropic media can be directly carried over.

The design principle is therefore to transform the constitutive parameters such that the electromagnetic fields are excluded from a specified region of space, and packed into another region in another geometry. An example was proposed in Pendry et al. (2006) for excluding a spherical cavity of electromagnetic fields and compressing these fields into a spherical shell of a medium enclosing the cavity. Consider the transformation

$$r' = a_1 + r(a_2 - a_1)/a_2, \qquad \theta' = \theta, \qquad \phi' = \phi \tag{10.1}$$

which takes every point $r < a_2$ into the region $a_1 < r < a_2$. Hence the field inside the spherical cavity $r < a_1$ and inside the spherical shell $a_1 < r < a_2$ are both compressed into the spherical shell. Application of the transformation relations Eq. (9.31b) indicates that

$$\varepsilon'_{r'} = \mu'_{r'} = \frac{a_2}{a_2 - a_1}\left(\frac{r' - a_1}{r'}\right)^2, \tag{10.2a}$$

$$\varepsilon_{\theta'} = \mu_{\theta'} = \frac{a_2}{a_2 - a_1}, \tag{10.2b}$$

$$\varepsilon_{\phi'} = \mu_{\phi'} = \frac{a_2}{a_2 - a_1} \tag{10.2c}$$

gives one set of parameters for $a_1 < r < a_2$ that can map the fields in the interior of the sphere into the spherical shell. Outside the spherical shell the material parameters take the values of unity. Note that at the outer edge of the spherical shell the material parameters of the shell satisfy

$$\varepsilon_{\theta'} = \varepsilon_{\phi'} = \varepsilon_{r'}^{-1}, \tag{10.3a}$$

$$\mu_{\theta'} = \mu_{\phi'} = \mu_{r'}^{-1}, \tag{10.3b}$$

which are precisely the conditions for a perfectly matched layer to vacuum (Berenger 1994). Thus, the spherical shell in this example is also perfectly

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impedance matched with free-space and hence no reflection or scattering arises at its interface. In this sense, invisibility is achieved in a similar manner to the impedance-matched absorber. However, in a major difference with this system, the spherical shell also contains within itself any fields that emanate from a source and smoothly guides the radiation away from the central core region. Furthermore, by construction this spherical shell does not affect the fields outside the shell. Thus, an observer cannot obtain any information about the concealed object or about the shell itself. Also note that the choices of the parameters for cloaking is not unique but that the above choices of parameters are the only ones that give perfect impedance matching as well.

Consequently, the shell of the anisotropic and inhomogeneous medium can enable perfect cloaking of an object located inside the spherical cavity. Note that conversely, the cloaked object inside the spherical cavity can never communicate with the external world because all the fields emitted by a source inside would be mapped onto regions within the cloak only. Thus the invisible object is also thus blind and secluded from the rest of the electromagnetic world at that frequency.

The main requirements of an invisibility cloak, namely, the anisotropy and inhomogeneity, are both well within technological reach. An experimental demonstration of cloaking of a cylinder using a system of split ring resonators was given in Schurig et al. (2006) where it was shown that the scattering from a cloaked cylindrical cavity was much reduced. Invisibility by a similar cloak was investigated in Zolla et al. (2007) using the finite element method and the response to a closely placed radiating line source was studied. It was found that the primary effect due to the invisibility cloak was that the objects near the cloak would appear slightly shifted from their original positions – in the manner of a mirage.

We also note that due to the necessary dispersion in the material parameters of metamaterials, the invisibility is principally functional only within a narrow band of frequencies. Another important limitation of such invisibility devices stems from the fact that in order to make the cloak layer substantially thinner than the cloaked region, materials with very large constitutive parameters are required. While obtaining such characteristics is feasible by working close to resonances, important challenges may arise, typically associated with dissipation.

10.3 Cloaking with localized resonances

A somehow natural idea when trying to reduce the scattering cross-section of an object is to look for a coating material that exhibits a compensating effect. For example, for a dielectric core inducing a positive polarizability, one

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could look for a shell that induces a negative polarizability. While this is a straightforward outcome in the context of complementary optical layers (see Section 8.2), it is not immediately straightforward in higher dimensions. By defining a proper metric (typically the scattering cross-section) and outlining a design procedure, one could adjust the parameters of the shell so as to cancel as much as possible the scattering of the core. Against intuition, one would thus realize a larger object, thus exhibiting a larger geometric cross-section, but a lower scattering cross-section. This is precisely the approach proposed by Alù and Engheta (2005).

Let us simplify the conceptual discussion to spherical geometries, for which the exact scattering field is known to be governed by the Mie theory (see Section 5.2.5 on 198). The scattering cross-section normalized to the geometric cross-section for a sphere of radius a is given by (Tsang et al. 2000a)

$$\sigma_{scat} = \frac{2}{(|k|a)^2} \sum_{m=1}^{\infty} (2m+1)(|a_m|^2 + |b_m|^2), \tag{10.4}$$

where k is the wavenumber of the surrounding medium and (a_m, b_m) are the coefficients of the external field expanded in the spherical coordinate system, as derived in Section 5.2.5. Minimizing σ_{scat} is seen to be a non-linear problem that does not offer a trivial solution. Let us therefore assume that the sphere is very small so that the quasi-static is applicable. Under this approximation, the small sphere responds primarily as a point source and thus re-radiates as a dipole. Under a $\hat{z}$ polarized incident field propagating along the $\hat{x}$ direction, the electric and magnetic fields can be written as (Kong 2000)

$$\mathbf{E}(\mathbf{r}) = -\frac{\mathrm{i}\omega\mu I\ell e^{\mathrm{i}kr}}{4\pi}\left[\hat{r}\left[\left(\frac{\mathrm{i}}{kr}\right)^2 + \frac{\mathrm{i}}{kr}\right] 2\cos\theta + \hat{\theta}\left[\left(\frac{\mathrm{i}}{kr}\right)^2 + \frac{\mathrm{i}}{kr} + 1\right]\sin\theta\right], \tag{10.5a}$$

$$\mathbf{H}(\mathbf{r}) = -\hat{\phi}\frac{\mathrm{i}kI\ell e^{\mathrm{i}kr}}{4\pi r}\left(\frac{\mathrm{i}}{kr} + 1\right)\sin\theta\,, \tag{10.5b}$$

where the dipole moment $I\ell$ can be determined by matching the boundary conditions. Under the quasi-static approximation, which is equivalent to a near-field approximation ($kr << 1$), we see that the electric field dominates over the magnetic field. Consequently, in order to reduce the scattering cross-section of such sphere, one only needs to design a shield that compensates for the first electric dipole term, all the other terms being negligible already.[‡]

[‡]Note that if the dipole term is canceled, the higher order terms become dominant and cannot be neglected anymore in the computation of the scattering cross-section. However, this scattering is already much weaker than the original one, which therefore goes along the purpose of reducing σ_{scat}.

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Within the Mie theory, the small sphere regime, also known as the Rayleigh regime, corresponds to reducing the sum to the first term $m = 1$ only. Under this approximation, it is seen that the TM term b_1' is dominant, confirming the fact that the response is primarily of electric nature (the prime indicates that the b_1' term is a generalization of the b_1 term of Eq. (5.33) to a two-layer spherical coating geometry). The reduction of the scattering cross-section is therefore reduced to the minimization of the b_1' term. This minimum has been shown to be exactly zero within the small sphere approximation if the ratio between the core radius r_c and the shell external radius r_s is (Alù and Engheta 2005)

$$\frac{r_c}{r_s} = \sqrt[2m+1]{\frac{(\varepsilon_c - \varepsilon_0)[(m+1)\varepsilon_c + n\varepsilon]}{(\varepsilon_c - \varepsilon)[(m+1)\varepsilon_c + n\varepsilon_0]}}, \tag{10.6}$$

where ε, ε_c, and ε_0 are the permittivities of the core, the shell, and the outside free-space, respectively. Upon studying this relation, it has been shown that in order to reduce the scattering cross-section of a regular dielectric core, one has to resort to shell permittivities that are lower than those of the outside medium (typically free-space), possibly taking negative values. This is in good agreement with the intuitive view of polarization compensation. As a matter of fact, the polarizations of the core and the shell can be written as $\mathbf{P}_1 = (\varepsilon - \varepsilon_0)\mathbf{E}$ and $\mathbf{P}_2 = (\varepsilon_c - \varepsilon_0)\mathbf{E}$, respectively, so that one is positive and the other one negative if $\varepsilon_c < \varepsilon_0 < \varepsilon$. In addition, although this condition has been derived within the Rayleigh approximation, it has been shown to still yield reasonably low scattering cross-sections for radii up to $\lambda/5$, i.e. away from the Rayleigh regime already.

Naturally, the same principle can be applied to the magnetic dipole, which can be reduced or even canceled by the proper choice of the permeability of the shell as function of the one of the core. In fact, in view of Eqs. (10.5), the magnetic dipole moment is the next important term after the electric dipole effect has been canceled. Consequently, further reduction of the scattering cross-section can be achieved by lowering the permeability of the shell simultaneously to lowering its permittivity.

Remarkably, this phenomenon of cancellation of scattering cross-section is not associated with a resonance of the Mie coefficients, which would instead yield a strongly enhanced scattering effect. Such resonances being nonetheless present, one has to be careful at not designing a configuration where the condition of zero and infinite σ_{scat} are too close to each other, in which case the sensitivity between transparency and strong scattering would be too large. In fact, whenever possible, it is judicious to design a configuration where no resonances are excited, which is possible by using only positive permittivities, with the internal one being lower than the external one (Alù and Engheta 2005).

The transparency achieved using this method presents one major advantage and one major disadvantage. The advantage is that since the cancellation of

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the scattering cross-section does not rely on any resonance, it is a robust effect to various perturbations. In particular, variations in sizes and material parameters, including losses, have been shown to have a moderate effect on the increase of the scattering cross-section, whereas variations due to geometrical imprecision in the realization of the optimized split-rings and rods (necessary to obtain the required effective constitutive parameters) have been speculated to have a limited impact as well, for the same reason. This weak sensitivity can also be translated into a robustness of the configuration to frequency changes, so that the technique presented here can be expected to perform relatively well over a frequency band surrounding the ideal situation.

This configuration, however, presents an important drawback which is that it is specifically optimized for a given core. In other words, the design and the physical implementation of the shell need to be repeated anew for each core property.

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