

**Physics and
Applications of**

**Negative
Refractive
Index
Materials**

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S. Anantha Ramakrishna
Tomasz M. Grzegorczyk

SPIE
PRESS

Bellingham, Washington USA



CRC Press

Taylor & Francis Group

Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business

Co-published by SPIE
P.O. Box 10
Bellingham, Washington 98227-0010 USA
Tel.: +1 360-676-3290
Fax: +1 360-647-1445
Email: Books@SPIE.org
spie.org
ISBN: 9780819473998

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

© 2009 by Taylor & Francis Group, LLC
CRC Press is an imprint of Taylor & Francis Group, an Informa business

No claim to original U.S. Government works
Printed in the United States of America on acid-free paper
10 9 8 7 6 5 4 3 2 1

International Standard Book Number-13: 978-1-4200-6875-7 (Hardcover)

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To Kanchan, Kartik, and Kanishka

To Alessandra, Eva, Davide, and to my parents

In memoriam: Jin Au Kong

Jin Au Kong, Professor in the Department of Electrical Engineering and Computer Science at the Massachusetts Institute of Technology (MIT), contributed tremendously to the development of left-handed media from the very beginning, in 2001. His influence can be felt in all areas of this field, from theory and numerical simulations for which he was internationally renowned, to experiments that he helped conceive and carry through all around the world, notably within MIT and the MIT Lincoln Laboratory, as well as in Asia and Europe. The conference he created, *Progress in Electromagnetic Research Symposium*, was one of the first, if not the first, to promote technical sessions on left-handed media, thus contributing to the cross-fertilization of ideas among researchers worldwide. His journals, the *Journal of Electromagnetic Waves and Applications* and the *Progress in Electromagnetic Research*, constantly call for innovative papers on all aspects of left-handed media, and have been well regarded and well cited. Finally, his textbooks, written and published years before the advent of left-handed media, often contain remarkable ideas and concepts that were later rediscovered and accepted as pillars in this new field.

Prof. Kong passed away unexpectedly on March 12, 2008, of complications from pneumonia, before this book went to press. Many of the ideas and concepts presented in the following pages have been directly inspired by him and discussed at length with him, often late into the night in his office at MIT. The course of events made it such that my efforts toward the realization of this book have become a tribute to his work during the last 7 years. The international community has lost one of its giants, which is in addition a personal loss for me.

T. M. Grzegorzcyk

Foreword

The past ten years have seen an astonishing explosion of interest in negative refractive index materials. First explored systematically by Veselago in 1968 from a theoretical point of view these materials remained without an experimental realisation for more than 30 years. That had to await development of suitable metamaterials, materials whose function is due as much to their internal sub wavelength structure as to their chemical composition. The added flexibility to create new materials enables properties unavailable in nature to be realised in practice. That opened the floodgates to a host of new experiments.

Why the great interest? From its rebirth at the beginning of this century negative refraction has provoked controversy. To be consistent with the laws of causality a material has to do much more than refract negatively. For example, it must necessarily be dispersive. Thus did many misunderstandings arise and pioneers had to endure some testing assaults. Yet even that aspect can now be seen as positive because controversy drew attention to the fledgling subject and showed that negative refraction contains subtleties that even experienced scientists did not at first appreciate. Even now we as a community are learning from our errors and discovering new aspects of this long hidden subject. As work progressed and news of amazing results spread beyond the scientific community into the popular press, a broader excitement has been generated. Some of the more extraordinary results such as the prescription for a perfect lens, and particularly the possibility of making objects invisible, had already been foreseen in science fiction and fed a ready-made appetite in the popular imagination. Thus the ancient subject of classical optics has brought us new discoveries and excitement.

This book, written by two leading practitioners of negative refraction, arrives at an opportune time because there is a substantial body of results available in the field that need to be gathered together in a systematic fashion sparing new arrivals hours of wasted time trawling through the very many papers in the literature. And yet new discoveries are continually reported. This is work in progress and the authors must steel themselves eventually to write a second edition!

Sir John B. Pendry
Imperial College London

Preface

Rarely in the history of science does one have the opportunity to witness an explosion of interest for a given topic, to participate in its development from its beginning, and to witness its growth at a pace almost exponential over a period of about a decade. Yet, we believe that this is precisely what has happened to us, with regard to the new development of materials that are now called metamaterials, left-handed media, or negative refractive media. Fundamentally rooted in the electromagnetic theory and governed by the equations proposed by the Scottish physicist James Clerk Maxwell at the end of the 19th century, the development of these structured composite materials that we call *metamaterials* could have been another incremental step in the more general research in electromagnetics and optics. Yet, the scientific community quickly realized that the implications and applications opened by the study of metamaterials are unprecedented, potentially revolutionary, and scientifically as well as technologically highly interesting and challenging. A new paradigm of electromagnetic and optical materials has evolved today from these studies.

The study of metamaterials is often thought of as being associated with negative refraction. It is much more than that. Over the past decade, scientists have shown how to manipulate the macroscopic properties of matter at a level unachieved before. For decades, our world was limited to materials with primarily positive permittivities and permeabilities, with some exceptions such as plasmas, for example, whose permittivities can be negative. The research in metamaterials coupled with the rapid advancements in micro- and nanofabrication technology has totally lifted this limitation, and has opened the door to almost arbitrary material properties with some extraordinary consequences across the electromagnetic spectrum, from radio frequencies to optical frequencies. This book is devoted to a discussion of these consequences as well as their theoretical implications and practical applications.

It is inevitable that such a growing field has attracted much attention in the scientific as well as in the more popular literature: the number of scientific articles has been in constant and almost exponential growth since about the year 2000, many popular articles have been published in scientific as well as nonscientific journals, while technical reviews and a few books have already been devoted to this field. It therefore appears ambitious at best and risky at worst to attempt the publication of an additional reference in this arena.

Nonetheless, we think that such an addition is necessary and was, in fact, missing. The extremely large number of scientific papers published is certainly vivid proof of the rapid evolution of this research area, but getting familiar

with and appreciating so much information also represent a daunting task for the student or researcher who is new to this field. In addition, the large number of new articles appearing on a weekly basis may also appear difficult to track, even by the expert researcher. It is with this spirit that we have targeted this book at as vast an audience as possible: the reader unfamiliar, but interested in this field, will find in the following pages the synthesis and organization of what we believe to be the most important and influential papers related to metamaterials, whereas the expert reader will hopefully find a useful viewpoint and detailed explanations of some of the most recent papers at the time of this writing, touching on as many aspects of this field as possible.

An additional motivation to undertake the writing of this book was our feeling that a coherent reference presenting the history, development, and main achievements of metamaterials was missing. Although some excellent books are already available to the reader, they are usually focused on either a very specific aspect of this field, or a compilation of chapters written by renowned scientists. In the present book, we have tried to remedy what we believe are limitations of the previous two formats by offering a book covering a wide variety of topics, yet having a coherence across chapters that enables the reader to cross-reference similar topics and, hence, to delve deeper into their presentation and explanation.

Naturally, it is impossible to present in a short book all aspects of a given scientific field, all the more when this field has become so vast and complex as the one the present book is devoted to. In addition, and despite our best efforts, our grasp of the field is also incomplete and is being refined by the day. We would therefore like to apologize upfront to those authors who may feel that their work is misrepresented or underrepresented in the following pages. May they put it on the account of our limited knowledge and not on our judgment of the quality of their contributions.

Finally, we must remark that it has been very difficult to write a book on an emerging area: it has almost been like writing about the personality of a growing teenager. New topics of today might disappear tomorrow or, instead, might reveal unexpected promises and become the front-runners of this research field. Metamaterials of the future will necessarily be robust and reliable, multifunctional, and reconfigurable to perform satisfactorily in various demanding environments. Today's metamaterials are quite primitive by these standards and developments are happening at breathtaking speeds. These have been the reasons why we decided not to have a concluding chapter – this book is an ongoing account of metamaterials.

S. A. Ramakrishna
Kanpur, India

T. M. Grzegorzcyk
Cambridge, Massachusetts, USA

Acknowledgments

This book came about not only because of our privilege to have witnessed the birth of this field, but more importantly because of our privilege to have actively participated in its development from a very early date. The research we have carried out over almost an entire decade brought us in contact with many researchers and students who, in many ways, have helped us discover and learn about this exciting topic. We would like in particular to acknowledge the contributions of our most closely related colleagues: Benjamin E. Barowes, Sangeeta Chakrabarti, Hongsheng Chen, Jianbing J. Chen, Xudong Chen (with a special thanks for proofreading parts of the manuscript), Sebastien Guenneau, Brandon A. Kemp (with a special thanks for proofreading parts of the manuscript), Jin Au Kong, Narendra Kumar, Akhlesh Lakhtakia, Jie Lu, Olivier J. F. Martin, Christopher Moss, Lipsa Nanda, Stephen O'Brien, Joe Pacheco, Jr., Sir John Pendry, Lixin Ran, Zachary Thomas, Harshawardhan Wanare, Bae-Ian Wu, and Yan Zhang.

We specifically thank L. Nanda and S. Chakrabarti for their help in making some of the figures and compiling the bibliography. We thank our colleagues from across the world who have given us permission to reuse or reproduce their figures and data which, at times, might have even been original and unpublished.

SAR acknowledges the support of the Centre for Development of Technical Education, IIT Kanpur via a book-writing grant and encouragement from his colleagues in the Physics Department at IIT Kanpur.

Finally, for their constant support and encouragements, we would like to thank our respective families to whom we dedicate this book.

About the authors

S. Anantha Ramakrishna received his M.Sc. in physics from the Indian Institute of Technology, Kanpur, and his Ph.D. in 2001 for his research work on wave propagation in random media at the Raman Research Institute, Bangalore. During 2001–2003 he worked with Sir John Pendry at Imperial College London on the theory of perfect lenses made of the newly discovered negative refractive index materials. In 2003, he joined the Indian Institute of Technology, Kanpur as an assistant professor and is presently an associate professor of physics there. His research interests concern complex wave phenomena in optics and condensed matter physics. He published the first comprehensive, technical review on the development of negative refractive index materials in 2005. He is a Young Associate of the Indian Academy of Science, Bangalore, a recipient of the Young Scientist Medal for 2007 of the Indian National Science Academy, Delhi, and was selected as an affiliate of the Third World Academy of Science, Trieste, in 2007. He was an invited professor at the Institut Fresnel, Université Aix-Marseille I in May 2006, and a visiting professor at the Nanophotonics and Metrology Laboratory at the Ecole Polytechnique Federale de Lausanne during June–July 2006. He is a member of SPIE and a life member of the Indian Physics Association.

Tomasz M. Grzegorzcyk received his Ph.D. from the Swiss Federal Institute of Technology, Lausanne, in December 2000. In January 2001, he joined the Research Laboratory of Electronics (RLE), Massachusetts Institute of Technology (MIT), U.S.A., where he was a research scientist until July 2007. Since then, he has been a research affiliate at the RLE-MIT, and founder and president of Delpsi, LLC, a company devoted to research in electromagnetics and optics. His research interests include the study of wave propagation in complex media and left-handed metamaterials, electromagnetic induction from spheroidal and ellipsoidal objects for unexploded ordnances modeling, optical binding and trapping phenomena, and microwave imaging. He is a senior member of IEEE, a member of the OSA, and was a visiting scientist at the Institute of Mathematical Studies at the National University of Singapore in December 2002 and January 2003. He was appointed adjunct professor of The Electromagnetics Academy at Zhejiang University in Hangzhou, China, in July 2004. From 2001 to 2007, he was part of the Technical Program Committee of the *Progress in Electromagnetics Research Symposium* and a member of the Editorial Board of the *Journal of Electromagnetic Waves and Applications* and *Progress in Electromagnetics Research*.

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