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The reflection and refraction of light across a material slab

In this Appendix, we will calculate the transmission or reflection coefficient from a slab of any arbitrary material. The material can be isotropic, anisotropic or bianisotropic. All that information is included in the Fresnel coefficients for reflection and transmission across the interfaces comprising the slab, the wave impedance in the different directions within the slab, and the dispersion relation within the medium via the component of the wave vector normal to the interfaces. The wave impedance is the same for an isotropic medium in all directions, but different for waves moving along the positive and negative directions as seen in Appendix B in a bianisotropic medium.

We will perform the calculations in three conceptually differen ways: (*i*) by the usual way of matching the tangential components of the fields inside and outside the slab, (*ii*) by using the fact that we have one upgoing and one downgoing wave in the medium, and (*iii*) by the multiple scattering method. The three methods obviously lead to the same final result which is a matter of consistency. But each of these methods can give rise to better understanding of the concepts involved and the resulting phenomena.

Method 1: Matching fields across the interfaces
Consider the material slab to occupy the region between the planes $z = 0$ and $z = d$ and the region outside to be vacuum. Consider a plane wave incident from the left to be incident on the slab $\exp(\mathrm{i}k_{z1}z$. The magnetic fields inside the slab medium can be written as

$$H_y = A\exp(+\mathrm{i}k_{z2}z) + B\exp(-\mathrm{i}k_{z2}z). \tag{C.1}$$

At $z = 0$ and $z = d$, we have the conditions of continuity for the tangential **E** and **H** fields as usual:

$$1 + R = A + B, \tag{C.2}$$
$$1 - R = \eta_p\ A + \eta_m\ B, \tag{C.3}$$
$$Te^{\mathrm{i}k_{z1}d} = Ae^{\mathrm{i}k_{z2}d} + Be^{-\mathrm{i}k_{z2}d}, \tag{C.4}$$
$$Te^{\mathrm{i}k_{z1}d} = \eta_p\ Ae^{\mathrm{i}k_{z2}d} + \eta_m\ Be^{-\mathrm{i}k_{z2}d}. \tag{C.5}$$

Solving the above, we have for the reflection (R) and transmission (T) coefficients of the slab

$$Te^{\mathrm{i}k_{z1}d} = \frac{2(\eta_p - \eta_m)e^{\mathrm{i}k_{z2}d}}{(1+\eta_p)(1-\eta_m) - (1+\eta_m)(1-\eta_p)e^{2\mathrm{i}k_{z2}d}}, \tag{C.6}$$

$$R = \frac{(1-\eta_p)(1-\eta_m)(1-e^{2\mathrm{i}k_{z2}d})}{(1+\eta_p)(1-\eta_m) - (1+\eta_m)(1-\eta_p)e^{2\mathrm{i}k_{z2}d}}. \tag{C.7}$$

In the case of an isotropic medium, we have $\eta_p = -\eta_m = \eta_i$ as defined in Appendix B. It can be easily verified that we have an invariance of these expressions when $k_{z2} \to -k_{z2}$ as we then also have $\eta_p \to \eta_m$ simultaneously. So the choice of the sign of the wave-vector inside the medium does not manifest in any measurable effect with a symmetric slab.

Method 2: Coupling waves across the interfaces

We note that we have the positive and negative waves coupled via the Fresnel coefficients to the incident, reflected and transmitted waves across the two interfaces at $z = 0$ and $z = d$ as

$$A = t_{pp} + r'_{pm}B, \tag{C.8}$$

$$Be^{-\mathrm{i}k_{z2}d} = r'_{mp}Ae^{\mathrm{i}k_{z2}d}, \tag{C.9}$$

$$Te^{\mathrm{i}k_{z1}d} = t'_{pp}Ae^{\mathrm{i}k_{z2}d}, \tag{C.10}$$

$$R = r_{mp} + t'_{mm}B. \tag{C.11}$$

Solving these equations for T and R, we get

$$Te^{\mathrm{i}k_{z1}d} = \frac{t_{pp}t'_{pp}e^{\mathrm{i}k_{z2}d}}{1 - r'_{pm}r'_{mp}e^{2\mathrm{i}k_{z2}d}}, \tag{C.12}$$

$$R = r_{mp} + \frac{t'_{mm}r'_{mp}t_{pp}e^{2\mathrm{i}k_{z2}d}}{1 - r'_{pm}r'_{mp}e^{2\mathrm{i}k_{z2}d}}, \tag{C.13}$$

which can be seen to be identical to the earlier one by substituting in the values of the Fresnel coefficients in terms of η_p, η_m and η_i presented before in Appendix B.

Method 3: The multiple scattering technique

Now let us consider the typical multiple scattering method in Born and Wolf (1999). The transmission and reflection can be written as a sum of infinite waves in terms of the partial Fresnel scattering coefficients:

$$T = t_{pp}t'_{pp}e^{\mathrm{i}k_{z2}d} + t_{pp}r'_{mp}r'_{pm}t'_{pp}e^{3\mathrm{i}k_{z2}d} + \cdots \quad , \tag{C.14}$$

$$R = r_{mp} + t_{pp}r'_{mp}e^{2\mathrm{i}k_{z2}d}t'_{mm} + \cdots \quad . \tag{C.15}$$

These geometric series can be summed to yield exactly the expressions given by Eq. (C.12) and Eq. (C.13), respectively, obtained in method 2. There is a crucial assumption that the optical path-lengths for forward-going and backward-going waves are the same. This is also true in the bianisotropic case, although the wave impedances in both directions are not the same.