A

The Fresnel coefficients for reflection and refraction

In this appendix, which is primarily meant for the uninitiated reader, we derive the Fresnel coefficients for the reflected and refracted radiation across an interface between two isotropic media with dielectric permittivities ε_1 and ε_2 and magnetic permeabilities μ_1 and μ_2. Consider Fig. A.1 for the geometry involved (the case of P-polarization is considered there).

The wave vectors in the two media then obey the dispersion (assuming that the plane of incidence is the x-z plane)

$$k_{1x}^2 + k_{1z}^2 = \varepsilon_1\mu_1\frac{\omega^2}{c^2}, \tag{A.1}$$

$$k_{2x}^2 + k_{2z}^2 = \varepsilon_2\mu_2\frac{\omega^2}{c^2}. \tag{A.2}$$

For proper continuity of the electromagnetic fields across the interface at all times, we require the frequency ω to be unchanged. Similarly, $k_{1x} = k_{2x}$ which is required to match the phases at all points along the x direction as the system is invariant along the x direction. This enables us to obtain the Snells law for the angle of incidence and refraction as

$$\pm\sqrt{\varepsilon_1\mu_1}\sin\theta_1 = \pm\sqrt{\varepsilon_2\mu_2}\sin\theta_2. \tag{A.3}$$

A proper choice for the sign of the square should be taken as per the discussion in Chapter 5. Now k_{1z} and k_{2z} can be obtained from the above dispersion relations.

For the P-polarized radiation, it is easier to work with the magnetic fields and for S-polarization it is easier to work with the electric fields. Note that the Fresnel coefficient would then differ by a factor of $\sqrt{\mu/\varepsilon}$ which is the impedance of the medium in the two cases. In the case of P-polarization, the magnetic field is normal to the plane of incidence. Hence, $\mathbf{H}$ has only a y component. Matching the tangential component of $\mathbf{H}$ across the interface, we have

$$H_{iy} + H_{ry} = H_{ty}, \tag{A.4}$$

where the subscripts $i,\ r,\ t$ stand for the incident, reflected and transmitted fields. The Maxwell equation,

$$\nabla\times\mathbf{H} = \frac{\partial\mathbf{D}}{\partial t}, \tag{A.5}$$

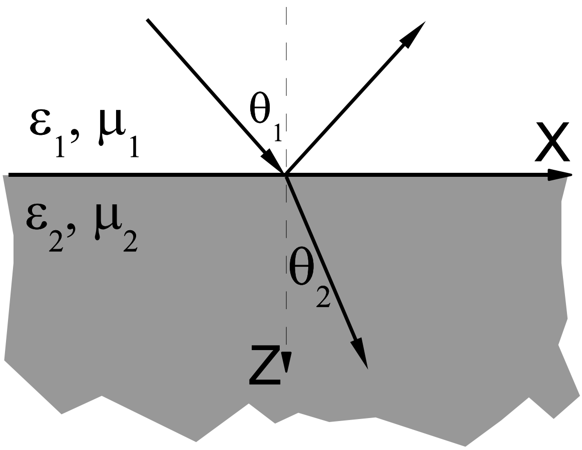

Figure A.1 The reflection and refraction of light across an interface between two isotropic media with arbitrary dielectric permittivity and magnetic permeability.

should be used to obtain the associated electric field. For a plane harmonic wave, we obtain

$$\mathbf{k} \times \mathbf{H} = -\omega\varepsilon\mathbf{E}, \tag{A.6}$$

from which we obtain

$$E_z = -\frac{k_x}{\omega\varepsilon} H_y, \tag{A.7}$$

$$E_x = \frac{k_z}{\omega\varepsilon} H_y. \tag{A.8}$$

The continuity of the tangential electric field (E_x) implies that

$$\frac{k_{iz}H_{iy}}{\omega\varepsilon_1} + \frac{k_{rz}H_{ry}}{\omega\varepsilon_1} = \frac{k_{tz}H_{ty}}{\omega\varepsilon_2}. \tag{A.9}$$

Noting $k_{rz} = -k_{iz}$ and eliminating H_{ry} from the two equations, we get

$$T = \frac{H_{ty}}{H_{iy}} = \frac{2k_{iz}/\varepsilon_1}{k_{tz}/\varepsilon_2 + k_{iz}/\varepsilon_1} \tag{A.10}$$

and similarly eliminating H_{ty} we obtain

$$R = \frac{H_{ry}}{H_{iy}} = \frac{k_{tz}/\varepsilon_2 - k_{iz}/\varepsilon_1}{k_{tz}/\varepsilon_2 + k_{iz}/\varepsilon_1}. \tag{A.11}$$

A similar analysis can be made for the S-polarized light. The results are similar with only ε being replaced by the corresponding μ everywhere and the coefficients relate the electric fields across the interface in this case.