# An electromagnetic model of the ball lightning

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A ball lightning model that can be imagined as a shock wave of a point explosion frozen with an internal strong laser discharge is considered.

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In the paper [1] it is claimed that electromagnetic models cannot explain observed parameters of ball lightnings, and should be rejected. Nevertheless, time to time such models are discussed in literature (see, for instance, [2]-[5]). Most of the authors, however, treat the ball ligtning as a plasma phenomenon. In particular, in [3] the ball lightning is supposed to be a cavity filled with radiation, which is locked by surrounding plasma.

We propose here a nonplasma model [6], which can be imagined as a shock wave of a point explosion in the atmosphere. This shock wave can be stopped and frozen by a powerful laser discharge behind the front of the wave, if the photons in the discharge experience total internal reflection at the interface of the shock wave front. In other words, the ball lightning is a spherical layer filled with electromagnetic radiation captured there because of the total internal reflection, and the layer itself does not dissipate because of electrostriction forces created by the radiation. This model contrary to [1] can explain both the high energy and the long life time of the ball lightning. More over it throws some light on other matural phenomena, such as hurricane and tornado and can be used in many other branches of physics. To show how the model works we use an analogy with quantum mechanics of a particle.

The stationary equation for electromagnetic potential (for simplicity we use here the scalar approximation)

$$[\Delta + n^2k^2]A(\mathbf{r}) = 0, \tag{1}$$

where $k = \omega/c$, $\omega$ is the photon frequency, $c$ is the light speed in vacuum and $n$ is the refraction index, is very similar to the Schrödinger equation

$$[\Delta + k^2 - u(\mathbf{r})]\psi(\mathbf{r}) = 0, \tag{2}$$

for particles [8]. Here $\psi$ is the wave function of a particle with wave number $k^2$, $u$ is a potential energy measured in units $\hbar^2/2m$ and $m$ is the mass of the particle. To transform equation (2) to the form (1) it is necessary only to denote

$$u = (1 - n^2)k^2.$$

It is known, that if $u$ in the equation (2) is a potential well, the particle can have bound states in it. Because of analogy of two equations (1,2) the same can be said also about the photons.

Now, let us consider interaction of matter with a particle. To be precise, we think of a slow neutron inside a substance. It is known (see, for example [7]) that the interaction of neutrons with matter is described by the potential $u = 4\pi N_0 b$, where $N_0$ is atomic density, i.e. the number of atoms in a unit volume, and $b$ is coherent scattering amplitude. If $b > 0$, then $u > 0$, and matter repels the neutron. However, if $b < 0$, the matter attracts it. Thus the substance with a negative $u$ represents a potential well for neutrons, and a neutrons inside such a matter can have bound states. If a neutron is in the bound state, the matter holds the neutron. However, and it is very important, not only the matter holds the neutron, the neutron itself also holds the matter, i.e it resists expansion of the matter and even compresses it, because the well depth depends on the atomic density $N_0$, and the higher is $N_0$ — the lower is the neutron's energy.

Indeed, it is shown in fig. 1 that, if the matter expands, the density $N_0$ and the well depth become lower. It leads to rise of the bound energy level, which requires supply of energy. In opposite case, the contraction of the well, and increase of the atomic density means lowering of the energy level, i.e to decrease of the neutron energy. The system of a matter with the bound neutron is unstable and tends to lower its energy, i.e. to compress the matter.

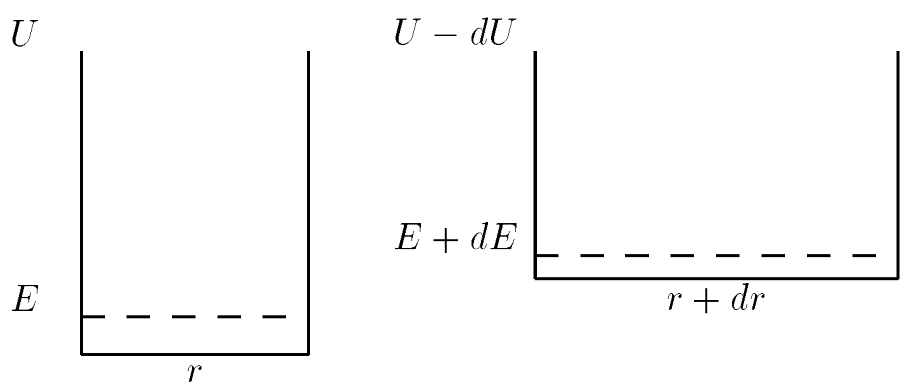


Fig.1

In a rectangular one-dimensional potential well the energy levels are determined by boundary conditions on the edges and infinity, which give the equation [8]

$$\cos(r\sqrt{U+E}/2) = \sqrt{(U+E)/U},$$

where $U$ is the absolute magnitude of the well depth ($U > 0$), and $r$ is its width. The compression force is

$$\frac{dE}{dr} = \frac{-E}{r} - \frac{(U+E)\sqrt{-E}}{2+r\sqrt{-E}}.$$

For low energy levels $E \approx -U$, and $dE/dr \approx U/r$.

The striction force created by a single particle is small. However if the number of bound particles is large, the force becomes also large. This may happen to be a very important factor for, say, neutron stars. In a neutron star every neutron has the potential $U = 4\pi N_0 b$, where $N_0$ is the density of neutrons, and $b$ is the neutron-neutron scattering amplitude. It follows, that the binding energy is proportional to $N_0^2$, and compression pressure (we call it "neutrostriction"), which is $p = -dU/dV$, where $V$ is the volume, can be very high. In some cases this contracting pressure can be even greater than the gravitational one.

We did not yet take into account that the neutron is a Fermi particle, and there is a repulsion between neutrons, because of the Fermi statistics [9]. However, even with account of the Fermi statistics, the neutrostriction pressure

$$p = -(\hbar^2/2m)[2\pi N_0^2|b| - (2/5)(3/8\pi)^{2/3}N_0^{5/3}]$$

at some densities $N_0$ is negative. It becomes negative at density of nutron matter of order $N_0 \approx 10^{30}$ cm$^{-3}$, if the amplitude $b$ of neutron-neutron scattering does not depend on energy,

The gravitational compression $p = -(4\pi/3)GR^2m^2N_0^2 \propto R^{-4}$ is proportional to $N_0^{4/3}$, thus for radius $R$ of the star, such that $(4/3)R^2m^3G/\hbar^2|b| < 1$, which corresponds to $R < 22.7$ km, the neutrostriction surpasses the gravitational compression. It means that at such a radius and for above density the star will be preserved even, if the gravity were suddenly switched off. For the star of the mass of our sun the density of neutrons at such radius is $N_0 \approx 2.5 \times 10^{37}$ cm$^{-3}$, and the depth of the potential well for a neutron is $U \approx 100$ MeV.

In the paper [5] the analogous interaction was considered for electrons. It was shown, that at some plasma density the attractive exchange interaction becomes larger than the Coulomb repulsion, which leads to coherent binding of plasma particles.

The notion of a potential well can be applied also to $\gamma$-quanta, and this leads to our model for the ball lightning. Indeed, for photons we use refraction index $n$, which is determined as $n^2 = \epsilon = 1 + 4\pi N_0\alpha$, where $N_0$ is the number of molecules in a unit volume and $\alpha$ is polarizability of a molecule. The refraction index gives an analog of photon-matter interaction potential

$$U = (1 - n^2)k_0^2 = -4\pi N_0 \alpha k_0^2,$$

which is negative for positive $\alpha$. It means that usually matter attracts photons.

It is easy to show, that one can attribute to a photon potential and kinetic energies inside a medium because of following reasons. The amplitudes of reflection, $r$, and transmission, $t$, for a photon of appropriate polarization at an interface of a medium are

$$r = (k_\perp - k'_\perp)/(k_\perp + k'_\perp), \quad t = 2k_\perp/(k_\perp + k'_\perp).$$

In the absence of absorption they satisfy the usual unitarity condition $|r|^2 + (k'_\perp/k_\perp)|t|^2 = 1$, which means conservation of the $\gamma$-particle. However the energy current, proportional to $\epsilon E^2/4\pi$, is not conserved:

$$(E^2/4\pi)|r|^2 + (k'_\perp/k_\perp)(\epsilon E^2/4\pi)|t|^2 > E^2/4\pi.$$

The energy conservation can be restored, only if the potential energy $(1 - \epsilon)E^2/4\pi$ is introduced. Then $\epsilon E^2/4\pi$ can be considered as the kinetic energy.

In negative potential photon can have bound states, which means that matter can hold photons. At the same time the photons also hold the matter. Indeed, the electromagnetic field interacts with an atom or a molecule via the potential

$$U_1 = -dE = -\alpha E^2 = -4\pi\alpha N_\gamma \hbar\omega,$$

where $d = \alpha E$ is the induced dipole moment (we suppose the atom has no own dipole moment), $\alpha$ is the polarizability of the molecule and $N_\gamma$ is the number density of photons. This interaction shows, that there is a force, which draws matter inside the photon field, and this force is proportional to the gradient of the photon density.

From quantum mechanics it follows [10, 11], that

$$\alpha = \frac{e^2}{2m_e}\sum_{k\neq 0}\frac{f_{0k}}{\omega_{0k}^2 - \omega^2 - \imath\omega\Gamma_k},$$

where $\omega$ is an incident photon frequency, $e$ is the electric charge, $m_e$ is the electron mass, $\omega_{kl} = \omega_k - \omega_l$ are eigen frequencies defined by transition $k \to l$, $\omega_k$ are energy levels of the electron in the atom, $\Gamma_k$ is the width of the transition, $f_{lk}$ are oscillator strengths given by

$$f_{kl} = (2m_e/\hbar^2)\hbar\omega_{lk}|d_{kl}|^2,$$

and $d_{kl} =< k|r|l >$ is a matrix element of the dipole transition between states $|l>$ and $|k>$.

For $\omega < \omega_{01}$ an unexcited atom is pulled into the space with larger density of photons. The matter-field attraction is known as "pondermotive" or "electrostriction" force. This force, for example, is responsible for self focussing of an intense laser beam. Here we shall show that this force, in analogy with quantum mechanics of a particle, leads to quasi bound states for photons, which is our representation of the ball lightning, and we shall estimate the life time and the total energy of such quasibound states and compare them with parameters of the ball lightning.

However atoms can also be repelled from the field. In particular, for the same condition: $\omega < \omega_{01}$, if the atom is excited, its interaction with the field is positive, and it is repelled from the space filled with photons. However, there is a probability, and we shall see it to be high, that the excited atom emits the photon and becomes pulled inside the photon gas. This is because the matrix element of the transition is proportional to the square root of the total number of the photons present in the mode. Because of this effect the photons in the intense coherent field can not be scattered or absorbed, and atoms in their turn become stabilized [12]-[14].

The matter-field interaction is positive also for free charges. This can be seen from plasma formula for refraction index $n^2 = 1 - \omega_0^2/\omega^2$, if $\omega^2$ is less than plasma frequency $\omega_0^2 = 4\pi N_e e^2/m$, where $N_e$ is density of charge $e$ and $m$ is its mass. It means that, if ionization happened at the explosion, then the light electrons will fly before the shock wave front, and heavy ions stay behind the front of the shock wave. After creation of spherical layer filled with intense electromagnetic field electrons remain to be separated from ions, and we obtain a charged spherical capacity.

Let us consider parameters of the ball lightning. We shall take radius and the energy of it to be given and equal to 10 cm and 10 kJ respectively and then estimate its life time.

In the spherical coordinate system the equation for radial part of the electromagnetic potential

$$A(\mathbf{r}) = \frac{R_L(r)}{r} P_L(\theta) \exp(\imath m\phi),$$

looks like

$$[\frac{d^2}{dr^2} + k^2 + u(r) - \frac{L(L+1)}{r^2}]R_L(r) = 0,$$

where $L$ is the orbital momentum of photons.

The total potential $L(L+1)/r^2 - u(r)$ is positive, and $u$ represents a "pocket" on a monotonously decreasing curve, representing the centrifugal potential. The photons have a metastable state in this pocket. Since the scattering is prohibited, the only way photons can leave this pocket is through tunneling.

Let the wave length of trapped radiation be $\lambda = 10^{-4}$ cm. Then, for sphere of radius $r_0 = 10$ cm we get $L = kr/2\pi \approx 10^5$. The life time $T$ can be estimated by expression $T = t_f/P$, where $t_f$ is free flight time between two collisions with the shock wave front and $P$ is a probability of tunneling through the potential barrier. Since $t_f < 10^{-10}$ s, the probability $P$ must be very low. Let us find $P$ with the usual quasiclassical approximation of quantum mechanics.

$$P = \exp(-\gamma), \qquad \gamma = \int_{r_1}^{r_2} \sqrt{L^2/r^2 - k^2}\, dr. \tag{3}$$

The integration limits are determined by the relations

$$(L/r_1)^2 = k^2 + u, \qquad (L/r_2)^2 = k^2.$$

At large $L$ and small $u$ the integral in (3) can be approximated by the expression

$$\gamma = \int_0^{x_2} \sqrt{u - 2L^2 x/r_1^3}\, dx = \frac{1}{3}\left(\frac{u}{k^2}\right)^{3/2} L,$$

where $x_2 = r_2 - r_1$. To get lifetime near $10^4$ s it is necessary to have $\gamma \approx 40$ and for $L = 10^5$ the value of $u/k^2$ should be $\approx 10^{-2}$. It gives the magnitude of the refraction index inside the shock wave to be of order 1.012.

The angle $\phi$ of total reflection is defined from $\sin\phi = 1/n$. It shows that the width of the photon layer is

$$d = r_0[1 - \sin\phi] \approx 0.01 r_0,$$

or $d \approx 0.1$cm. All these parameters are not extraordinary, so the life time of order 10 000 seconds seems to be quite achievable.

If total energy is concentrated in photons, then the layer must contain $10^{23}$ photons of energy 1 eV each. The density in the layer then is equal $N_\gamma \approx 10^{27}$ m$^{-3}$. At such a density the surface tension is $\sigma = (n^2-1)\hbar\omega N_\gamma d \approx 10^3$ J/m$^2$, so the gas density inside the ball can be only 20% higher than outside or the temperature inside gas is only 60 K higher than outside. Because of higher density inside the photon film, the ball is heavier than environment and falls down. If the gas density in the ball is lower than outside, its temperature can be higher, and the ball can be lighter than the air.

For photon frequencies very close to a resonance the magnitude of $n^2 - 1$ can be higher, and higher can be the surface tension and gas temperature inside the ball. Situation improves even more if one takes into account the Lorenz-Lorentz correction.

The energy of the ball contains also the energy of the spherical capacitor and the last depends on its charge. Let us suppose that the charge is equal to $Q$. An outside electron is attracted by the charge of ions with the force $F_q = Ee = 9 \times 10^9 Qe/r_0^2 = Q \times 10^{-7}$ N. But the photons repel it. The interaction energy of an electron with the photon layer is

$$u_e = (e^2/mc^2)\lambda^2\hbar\omega N_\gamma, \tag{4}$$

The repulsive force is proportional to the gradient of $N_\gamma$. The distribution of gamma quanta is determined by the Bessel function $J_L(kr)$, so $dJ_L(kr)/dr \approx (L/r_0)J_L(kr)$. It means that the force can be estimated as $F_e = Lu_e/r_0$, or $F_e \approx 10^{-12}$ N. This force can withstand the attraction only if the charge is $Q \leq 10\,\mu$Coul. So the total energy of the capacitor is of the order of 1 J, which is considerably smaller the total energy. But this is true for a single electron. For a negative ion the potential (4) can be two order of magnitude higher, and it increases $Q$ and its electrostatic energy.

To create the ball lightning it is necessary to make a point explosion inside a medium, where the shock wave makes excitation of atoms. Also it is possible to use an external pumping. The question is whether the laser discharge will have enough time to be developed.

To answer this question we compare the time of the light passage around the ball with that of the shock wave passage over the distance $d$. The first time is equal to $T_l = 2\pi r_0/c \approx 10^{-9}$ s. The second one, $T_s$, is defined by an automodel solution [15]:

$$r = (t^2 W/\rho)^{1/5},$$

where $\rho$ is the density of the atmosphere and $W$ is the energy of the explosion. The speed of the front is equal to

$$v = dr/dt = (2/5)r/t = (2/5)(W/\rho)^{1/2}r^{-3/2}.$$

At $W = 10^4$ J, $\rho = 1$ kg/m$^3$, $r = 0,1$ m the speed is $v \approx 10^3$ m/s. So the $T_s \approx 10^{-6}$ s. It shows that the laser discharge has enough time to be developed.

It is not necessary that each point explosion will lead to a ball lightning. Probability of the ball creation is proportional to the probability of emitting a photon in the good mode. In principle it is possible to stimulate laser discharge using, for instance, down conversion of external beam of photons.

It is interesting that external excited atoms incident on the ball may be reflected or deexcited. The last channel is the most probable. After deexcitation the photon layer pulls the atom inside it, so the ball moves in the direction of the positive gradient of the density of excited atoms and eats them up.

In fact, to have a stable ball it is not necessary that all photons inside the ball skin must be coherent. The coherence is necessary for fast process. If time scale is large enough we can obtain a similar object with incoherent radiation. It is possible that the origin of hurricanes and tornado is also connected with similar processes (see also [16]).

Till now we considered the scalar case. A spherical solution for vector electromagnetic field [17] in a reference frame moving with a small velocity $k$ can be represented, for instance, in the form

$$\mathbf{E} = C\exp(i\mathbf{kr} - i\omega t)\times$$

$$\times\left[\sqrt{\frac{l}{2l+1}}\mathbf{Y}_{l,l+1,M}(\mathbf{r}')j_{l+1}(s|\mathbf{r}-\mathbf{k}t|) + \sqrt{\frac{l+1}{2l+1}}\mathbf{Y}_{l,l-1,M}(\mathbf{r}')j_{l-1}(s|\mathbf{r}-\mathbf{k}t|)\right],$$

similarly to nonspreading wave packet in quantum mechanics [18], where $\mathbf{Y}_{j,l,M}(\mathbf{r}')$ is a vector spherical harmonics, and $\mathbf{r}' = (\mathbf{r}-\mathbf{k}t)/|\mathbf{r}-\mathbf{k}t|$.

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