PHYSICAL REVIEW E VOLUME 62, NUMBER 5 NOVEMBER 2000

Ball lightning as a force-free magnetic knot

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(Received 24 March 2000; revised manuscript received 3 May 2000)

The stability of fireballs in a recent model of ball lightning is studied. It is shown that the balls shine while relaxing in an almost quiescent expansion, and that three effects contribute to their stability: (i) the formation in each one during a process of Taylor relaxation of a force-free magnetic field, a concept introduced in 1954 in order to explain the existence of large magnetic fields and currents in stable configurations of astrophysical plasmas; (ii) the so called Alfven conditions in magnetohydrodynamics; and (iii) the approximate conservation of the helicity integral. The force-free fields that appear are termed ''knots'' because their magnetic lines are closed and linked.

PACS number(s): 52.80.Mg, 47.65.+a

I. INTRODUCTION. BALL LIGHTNING

This intriguing natural phenomenon consists of fireballs that sometimes appear near the discharge of a normal lightning, maintaining their brilliance, shape, and size up to 10 s or even more. After that, most end their lives smoothly, others with an explosion. Typically, their diameter is in the interval 10–40 cm, and their radiance is less than 150 W. A number of explanations for them have been proposed, but no one is generally accepted [1–7]. In this paper we develop some aspects of a model proposed by the authors (to be called the topological model, or just the model, in the following).

Several properties of ball lightning are very difficult to explain. First is their surprising stability and long lifetime. Second, since they emit light, it can be expected that something is hot inside, but hot air expands and moves upward, while ball lightning does not seem to change its size and has a clear tendency to move horizontally. Third, there is a curious contradiction in witness reports. Some claimed that ball lightnings is cold since they did not feel heat when it passed nearby, but others stated that ball lightning is surely hot since they were burned and had to receive medical attention after touching it, fires having also been produced in some cases.

These three difficulties seem to indicate that some unknown stabilizing mechanism acts in fireballs, producing some kind of effective cohesive force. Their appearance near lightning bolts gives strong support to the assumption that fireballs are an electromagnetic phenomenon with plasma and a magnetic field inside them. However, two serious objections have been raised against this idea: the problem of the output and the problem of the equilibrium. The first is that a ball of hot plasma with the observed dimensions would radiate with a power on the order of 1 MW or more, at least five orders of magnitude too much. The second objection is that as witnesses did not report changes in their radii, the balls seem to be in stationary equilibrium; however, no electromagnetic model with a suitable equilibrium configuration has been ever found, despite of much effort, because the magnetic pressure would make it unstable, causing an explosion. Indeed, this argument has a prestigious tradition, since Faraday himself argued that ball lightning cannot be an electric phenomenon because no electric configuration can remain in equilibrium for such a long time, this being one basis for some people's belief that it is just an optical illusion. Later, the virial theorem was used to rule out such electromagnetic models in which the balls are in equilibrium.

The organization of this paper is as follows. In Sec. II, we review the basic ideas of a model of ball lightning proposed by the authors. The concept of magnetic knots and of a force-free field are introduced in Secs. III and IV. The Taylor relaxation process is described in Sec. V. Sections VI and VII deal with the formation and evolution of fireballs in a topological model. Section VIII discusses why and how an electromagnetic model of ball lightning is possible, studying the reasons for the stability and slowness of the expansion of the fireballs. The good agreement of the predictions of the model with the observations, as reported by the witnesses, is explained in Sec. IX, and Sec. X summarizes the results.

II. TOPOLOGICAL MODEL

This paper discusses a recent topological model of ball lightning that describes this phenomenon as a system consisting of two subsystems in interaction: a magnetic field, with its magnetic lines linked to one another, and a set of linked streamers containing a plasma of ionized air. The first version [8], in which all the ball is ionized, was proposed in

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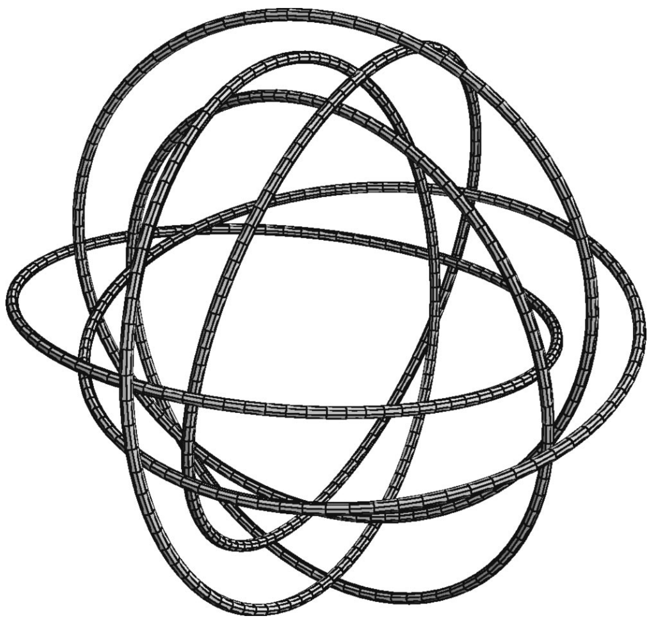

FIG. 1. Schematic aspect of several magnetic lines of a magnetic knot. Any two of the six lines shown are linked once. The same drawing also serves as a representation of the streamers along which electric currents flow inside a fireball in the topological model. Note that the hot plasma is confined in a set of linked streamers like those represented here, its relative volume ξ being small, the rest of the ball being at ambient temperature.

1996, and showed something interesting: the linking of plasma streamlines and magnetic lines has a stabilizing effect, giving a clue as to the long lifetime of the balls. In other words, the topology of the lines, both magnetic and of current, has a strong effect in the stability of the system. An important point is that in this model, if the so called Alfven conditions between the magnetic field and the fluid velocity and pressure are verified, the system is stationary in the magnetohydrodynamics (MHD) approximation; however it cannot be so in the exact theory, since it can lower its energy by expanding. However, this first version was too simple and had two drawbacks: the radiated power was too high and the ball expanded more than what the witnesses reports allowed. A second version [9] proposed in 1998 was more realistic. It assumed the following

(i) Only a very small part of the fireball consists of plasma of ionized air (on the order of 10^{-6} of the volume for the average ball), this explaining why its overall radiation is low, similar to that of a home electric bulb.

(ii) This plasma is confined inside closed streamers along which electric currents flow; these streamers are linked, like those represented in Fig. 1.

(iii) A magnetic field with linked lines is coupled to the streamers.

The agreement of the model predictions with witness reports is striking. However, the model was presented in Ref. [9] by means of particular examples. Here we give a formulation of general validity that is free from this restriction. It is also based on assumptions (i)–(iii), and offers a physical picture for the formation, evolution, and death of the fireballs.

As ours is an electromagnetic model, it must meet the two objections against that kind of model explained in Sec. I. As for the brilliance, since the streamers occupy only a small fraction of the ball volume (of the order of one part per million in the average case) the problem of the radiation is solved: in fact the model predicts outputs of the order of 10–150 W, in agreement with the reports.

Concerning the equilibrium problem, the fireballs are not stationary in the model but in expansion (they shine during their relaxation to a minimum energy state). However, this is a slow expansion, which can be qualified as almost quiescent, in which the radius increases at a slow pace, difficult to perceive by an excited witness, but nevertheless progressive.

As will be shown in Sec. VIII, the electromagnetic diffusion of the magnetic field and the current (that would otherwise destroy the structure) is hindered by the low temperature of the air between the streamers. Indeed the air must be heated in order to become a conductor, and this takes time. In this sense, our fireballs are not purely electromagnetic phenomena but are submitted to thermodynamical considerations. This is why the virial theorem does not affect this model, since it cannot preclude such behavior.

Indeed, this paper gives a sounder foundation to the second version of the model, by showing that its stability properties can be understood as a consequence of several effects. One is (i) the relaxation of the magnetic field to a force-free configuration, a concept introduced in 1954 in order to allow large currents and magnetic fields to exist in astrophysical plasmas [10]. This is curious, since it shows that an idea taken from astrophysics can be applied here on Earth. Others are (ii) that some solutions for the magnetic field and the plasma motion obey the so called Alfven conditions, under which the balls would be stationary in the MHD approximation neglecting radiation (although they are not so in the exact theory); and (iii) that conservation of the helicity integral. Assuming that the average magnetic field inside the fireballs is in the range 0.5–0.7 T (a normal value around lightning discharges), the predictions on brilliance, radius, energy, and lifetime agree with the values observed by the witnesses. It must be stressed that it is enough for the validity of the model that these three effects hold in an approximate way.

A warning is necessary here. The model uses streamers that have short circuited to form closed loops of current. Although this is perhaps not widely known and might seem strange, closed loops were observed in fact by Alexeff and Rader in a beautiful experiment [11] in which they produced high voltage discharges and observed that above about 10 MV numerous closed loops were formed. They stated that ‘‘they may be precursors of ball lightning’’ and that ‘‘the loops contract and quickly become compact force-free loops that superficially resemble spheres.’’ Although they did not consider the possibility of linked loops, such as those that we use in our model, we can safely assume that, in a certain small fraction of cases, some streamers can close as linked loops under the strongly stochastic conditions around a discharge. In fact, as shown in Ref. [9], closed loops of current have very surprising properties.

III. MAGNETIC KNOTS

The term ‘‘electromagnetic knot’’ was coined in Ref. [12] to denote a class of electromagnetic fields, solutions of the standard Maxwell's equations, with very curious and intriguing properties. They are defined by the condition that their force lines are closed curves and that any pair of magnetic lines, or any pair of electric lines, is a link. This means that, given any pair of magnetic (electric) lines, each one of them turns around the other a certain fixed number of times n_m (n_e). In this paper we consider only the case of magnetic knots, (i.e., with a vanishing electric field $\mathbf{E}=\mathbf{0}$), characterized by the linking number n_m of any pair of magnetic lines (noted simply as n in the following), which have the aspect shown in Fig. 1. The electromagnetic fields usually considered have unlinked lines, but those with linked lines have very interesting and appealing properties, the reader being referred to Refs. [12–14], where these electromagnetic knots were studied in detail.

Following the method explained therein, a magnetic knot can be built by means of a scalar function $\phi(\mathbf{r})$ that is constant along the magnetic lines. An important quantity in this context is the *magnetic helicity*, defined as

$$h=\int_{R^3}\mathbf{A}\cdot\mathbf{B}d^3r, \tag{1}$$

where $\mathbf{B}$ and $\mathbf{A}$ are the magnetic field and its vector potential. It is easy to show that this integral gives a measure of the curling of the magnetic lines to one another, this being the reason for its name [15]; thus it cannot vanish if the lines are linked [16]. Conversely, the lines are linked if $h\neq 0$.

We are interested in this paper in the case of a weakly resistive plasma in the MHD approximation. The following equation

$$\eta\mathbf{j}=\mathbf{E}+\mathbf{v}\times\mathbf{B} \tag{2}$$

is then verified, η being the resistivity, $\mathbf{j}$ the current density, and $\mathbf{v}$ the fluid velocity. By taking the time derivative of Eq. (1), assuming that the field goes to zero at infinity (i.e., outside the ball), it follows that

$$\frac{dh}{dt}=-2\int\mathbf{E}\cdot\mathbf{B}d^3r=-2\int\eta\mathbf{j}\cdot\mathbf{B}d^3r. \tag{3}$$

If the product $\eta\mathbf{j}\neq 0$, h is not conserved, in some cases because the lines may lose their individuality as they break and reconnect. Note, however, that, if $\eta\mathbf{j}=0$, h is a conserved quantity, even if one of the two factors is nonvanishing at any point. This last remark will be important later, in Secs. VII and VIII.

The magnetic helicity is important in the study of tokamaks and astrophysical plasmas. The same idea appears in fluid dynamics in a different form but with similar properties, as $h=\int\mathbf{v}\cdot\boldsymbol{\omega}d^3r$, $\mathbf{v}$ and $\boldsymbol{\omega}$ being the velocity and the vorticity (see, for instance, Refs. [17,18]; in fact, the term helicity was coined by Moffatt in this context [15]).

A property of integral (1) will be important later. Because of dimensional reasons, the magnetic field of a time independent knot can always be written as

$$\mathbf{B}=\frac{b}{L^2}\mathbf{f}\left(\frac{\mathbf{r}}{L}\right), \tag{4}$$

where L is a length scale, $\mathbf{f}$ a dimensionless vector function, and b a normalization constant with dimensions of magnetic field times square length [14]. The helicity integral is invariant under scale dilatations given by changes in L. Inserting Eq. (4) into Eq. (1), it is easily seen that h does not depend on L.

IV. FORCE-FREE MAGNETIC FIELDS

This concept was introduced in 1954 by Lust and Schlute [10] to explain the stability of astrophysical plasmas. A force-free magnetic field is defined by the condition

$$(\boldsymbol{\nabla}\times\mathbf{B})\times\mathbf{B}=0 \tag{5}$$

in the MHD approximation [19], which means that the magnetic force on the current vanishes. This is a very important idea to understand the evolution of a system with linked magnetic lines and linked streamers, as we will see in the following. Chandrasekhar and Woltjer showed a long time ago that force-free fields are among the fields with maximum magnetic energy for a given mean square current density [20]. In other words, they can sustain large magnetic energies. In a MHD approximation with infinite conductivity, Woltjer showed the same year that ''force-free fields represent the lowest state of magnetic energy which a closed system may attain.'' As we have seen, the helicity integral must be conserved in this case, so that he looked for the minimum of the magnetic energy with that constraint, introducing the corresponding Lagrange multiplier λ. The variational problem is then

$$\delta\int d^3r[(\boldsymbol{\nabla}\times\mathbf{A})^2-\lambda\mathbf{A}\cdot(\boldsymbol{\nabla}\times\mathbf{A})]=0, \tag{6}$$

the solution verifying

$$\boldsymbol{\nabla}\times\mathbf{B}=\lambda\mathbf{B}, \tag{7}$$

with constant λ. We see that the solution is a force-free magnetic field. Intuitively, we can say that, as the Lorentz force vanishes, the magnetic energy must be a minimum, since it cannot be transformed into motion energy of the plasma.

Some time later, Voslamber and Callebaut [21] provided an important precision by showing that (i) what had been proved really was just that all the extrema of the energy functional of a magnetic field coupled to a plasma are force free (and, vice versa, that force-free fields give extrema); but (ii) these extrema are not necessarily minima: there are some exceptions which can lead to instabilities. Nevertheless, two properties are still valid and must be retained: (a) all the minimum energy states are force-free fields, and (b) ''force-free fields may contain a huge amount of energy'' [22].

To summarize the results of Chandrasekhar and Woltjer and Voslamber and Callebaut, a magnetic field coupled to a plasma decays to a minimum of the energy, which has a force-free configuration. This final state is stable because, as the magnetic force on the current vanishes, the system cannot lose energy by rearranging its streamlines. The relevance of these ideas to ball lightning is clear if we accept that there is a magnetic field inside. Indeed, the main obstacle to ball lightning theory is to account for its surprising stability.

Force-free fields have an interesting property with pertinent consequences. Let us consider a force-free magnetic knot coupled to a plasma. In the magnetohydrodynamical MHD approximation, the motion inside the streamers is described by the Navier-Stokes equation coupled to the Maxwell equation for the magnetic field. If $\mathbf{v}$ is the plasma velocity, p the pressure and ρ the density, these equations are

$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}=-\frac{1}{\rho}\nabla\left(p+\frac{B^2}{2\mu_0}\right)+\frac{1}{\mu_0\rho}(\mathbf{B}\cdot\nabla)\mathbf{B}, \quad (8)$$

$$\frac{\partial \mathbf{B}}{\partial t}=\nabla\times(\mathbf{v}\times\mathbf{B})+\frac{1}{\sigma\mu_0}\Delta\mathbf{B}, \quad (9)$$

where $\mu_0=4\pi\times10^{-7}$ Wb/A m is the vacuum magnetic permeability, and σ the conductivity. If $\sigma=\infty$, the following is a stationary solution of the system of equations (8) and (9):

$$\mathbf{v}=\pm\frac{\mathbf{B}}{\sqrt{\mu_0\rho}}, \quad p+\frac{B^2}{2\mu_0}=\text{const.} \quad (10)$$

[Conditions such as Eq. (10) on the solutions were first considered by Alfven in 1942, when studying hydromagnetic waves.] The last term in Eq. (9) produces a difusion of $\mathbf{B}$ if the conductivity is finite. It will be seen that its effect becomes progressively more important along the life of the fireball, as the resistivity increases.

Because in a force-free magnetic field $\mathbf{B}$ and $\mathbf{j}=\nabla\times\mathbf{B}/\mu_0$ are parallel, the first Alfven condition [Eq. (10)] states that the velocity and current are parallel in the MHD approximation. This property will be important later: *in a force-free magnetic field the Alfven conditions imply that both the electron and the ions move along the magnetic lines* (in opposite directions). We will assume in this work that conduction inside the balls proceeds along streamers, which will carry positive and negative charges along the same channels. Note also that these streamers cannot be cut by the pinch effect, since the Lorentz force vanishes in a force-free magnetic field.

To end this section, two remarks are in order. First, the final state with a force-free configuration has a finite minimum energy if the system is inside a container. If this is not so, the final relaxed state has zero energy (note that in astrophysical applications the containment is often provided by the gravity). As will be explained in Sec. V, we assume in our model that the balls first reach the force-free configuration at a finite radius, and thereafter continue to decrease the energy by expansion and radiation.

Second, the radius of the balls L must be defined as that of the smallest sphere that contains all the streamers, since it coincides with the bright region. Obviously, the magnetic field extends farther than L, going to zero at infinity. Because of Eq. (9), the streamers are stationary in the ideal MHD approximation if the Alfven conditions [Eq. (10)] are verified along them (for $r<L$). The magnetic field must also be stationary in this approximation, as it is ''attached'' to the streamers (in the sense that $\nabla\times\mathbf{B}=\mu_0\mathbf{j}$). However, as will be seen, the system of streamers and magnetic field cannot be in a stationary state in the exact theory, since it can lower its energy by expanding its radius L. But this shows still that the Alfven conditions have a stabilizing effect on the system, even if they hold only in an approximate way inside the sphere of radius L, and notwithstanding the fact that the gradient of magnetic pressure is high at some places for $r>L$ (where B decreases quickly).

V. TAYLOR RELAXATION

The problem of evolution toward the relaxed final state with minimum magnetic energy was solved by Taylor [23]. He considered a plasma as a conducting fluid with small resistivity and viscosity. Even with these simplifying assumptions, its interaction with a magnetic field is very complex, especially if there is turbulence. Nevertheless, it is possible to give predictions about the plasma behavior, because the combined effect of the turbulence and the resistivity, even if small, is to dissipate energy, allowing the plasma to reach a state of minimum energy, ''the relaxed state,'' in a process taking place in a time shorter than the usual resistive time. Taylor developed the theory of this relaxation [23] and applied it successfully to diverse situations, including tokamaks and astrophysical plasmas.

A perfectly conducting plasma can be understood as an infinity of intertwined flexible conductors. The energy must be minimized under adequate constraints. With no constraints, the minimal energy state would be a vacuum field without current. However, if the plasma is a perfect conductor, $\eta=0$, there is an infinity of constraints: the fluid moves in such a way that each line maintains its identity (no breaking or reconnection of lines), the strength of any magnetic tube being constant. In this case, one has

$$\mathbf{E}+\mathbf{v}\times\mathbf{B}=\mathbf{0}, \quad (11)$$

which leads to

$$\frac{\partial \mathbf{A}}{\partial t}=\mathbf{v}\times\mathbf{B}+\nabla\chi, \quad (12)$$

χ being a scalar potential. Let $\mathbf{A}_\perp$ and $\mathbf{A}_\parallel$ be the components of $\mathbf{A}$ normal and parallel to $\mathbf{B}$. It is clear that a change in $\mathbf{A}_\perp$ can be absorbed in a redefinition of $\mathbf{v}$, so that Eq. (12) imposes a constraint on $\mathbf{A}_\parallel$, although not on $\mathbf{A}_\perp$, since it implies

$$\mathbf{B}\cdot\nabla\chi=\mathbf{B}\cdot\frac{\partial \mathbf{A}}{\partial t}. \quad (13)$$

A convenient way to express these constraints is to divide the volume in infinitesimal tubes surrounding closed magnetic lines, and stating that the quantities

$$h(\alpha,\beta)=\int_{\alpha,\beta}\mathbf{A}\cdot\mathbf{B}d^3r \quad (14)$$

are invariant (α and β labeling the magnetic line). The effect of this infinity of constraints is that the linking number of any pair of lines does not change in a perfectly conducting plasma. Now, to minimize the magnetic energy,

$$W=\frac{1}{2}\int(\nabla\times\mathbf{A})^2 d^3r, \tag{15}$$

submitted to constraint (14), a Lagrange multiplier $\lambda(\alpha,\beta)$ must be introduced. It then turns out that, for a perfectly conducting plasma, the equilibrium state satisfies

$$\nabla\times\mathbf{B}=\lambda(\alpha,\beta)\mathbf{B}, \tag{16}$$

where λ is a certain function verifying $\mathbf{B}\cdot\nabla\lambda=0$. Note that Eq. (16) proves that $\mathbf{B}$ is a force-free magnetic field.

However, there is a problem because, in order to determine the Lagrange multiplier, the invariants $h(\alpha,\beta)$ have to be calculated first, this implying that the final state (16) is not independent of the initial conditions. This would not be a relaxation process.

We escape from this problem taking into account that the conductivity of a real plasma is not infinite. This is important because the topology of the force lines does change in the presence of resistivity, however small: the magnetic lines break and reconnect. This happens even if the resistive diffusion time is long and the flux dissipation is small. The consequence of this is that, in a resistive and turbulent plasma, the magnetic tubes do not maintain their individuality, the topological invariants $h(\alpha,\beta)$ no longer being useful because it is not possible to keep the label (α,β) of the lines during the entire relaxation process. Nevertheless, the addition of all the invariants, which is equal to the helicity integral $h=\int\mathbf{A}\cdot\mathbf{B}d^3r$, is still a good invariant as long as the resistivity is small.

In order to obtain the relaxed state in a weakly resistive plasma, Taylor minimized the magnetic energy, taking as the only constraint the invariance of the total magnetic helicity [Eq. (1)], the integral being extended to all the volume occupied by the plasma. He found that the magnetic field satisfies

$$\nabla\times\mathbf{B}=\lambda\mathbf{B}, \tag{17}$$

where λ is now a constant uniquely determined by the helicity and the total flux (in a torus, this would be the toroidal flux). What is important here is that the final relaxed state is a force-free magnetic field that cannot dissipate any more energy through the action of the Lorentz force. It is true that the Lorentz force does not work over a particle in empty space, but dissipates energy by moving the current of lines. To understand this point, let us imagine the currents as flexible conductors in a viscous medium, as suggested by Taylor. But the system can still lower its energy by radiation.

As a final comment for this section, it must be remarked that Taylor developed his model for systems in a container. If there is no boundary, the system must relax to zero energy, expanding to an infinite radius. We assume in this work that the force-free condition is reached first at a finite radius L_0, the expansion going on afterward.

VI. FORMATION OF THE FIREBALL

It must be remembered that air does not conduct as a continuous medium. Quite the contrary, lightning or arc discharges proceed along lines well defined and separated from one another, the so called streamers, which are very narrow channels where the air is highly ionized, the charges moving along them with great mobility [25,26]. They are indeed thin tubes of highly conducting plasma. As a consequence of the previous considerations, the formation of the fireball in the topological model would consist in two steps: linking of the lines and relaxation to a force-free configuration.

(1) *Linking of the lines*: Near the discharge of ordinary lightning, where air is ionized and many currents along streamers are formed, the joint effect of powerful electric and magnetic fields may cause some streamers to short circuit and link to one another, generating closed loops, which behave as highly conducting linked coils (let us stress that, as indicated above, closed streamers is an observed phenomenon [11]). The magnetic lines are also linked, the system being characterized by the nonvanishing value of the magnetic helicity.

(2) *Relaxation to a force-free configuration*: Along a process similar to the Taylor relaxation described in Sec. V (with the only difference that the current flows along well separated streamers), a state is formed very rapidly that consists in a force-free magnetic knot coupled to the plasma inside the streamers. The plasma is hot enough to assume that the nonvanishing helicity integral is conserved (as has been explained, and will be discussed further in Sec VIII). As shown at the end of Sec. IV, because of the force-free condition $(\nabla\times\mathbf{B})\times\mathbf{B}=\mathbf{0}$ and the Alfven condition, the magnetic field is parallel to the current in such a way that ions and electrons move along the same streamers in opposite directions. Consequently, the streamers and magnetic lines have the same linking numbers, both having the aspect of the lines in Fig. 1. *The formation of this very tangled structure marks time zero*. Let ξ be fraction of the ball volume V occupied by the plasma (i.e., the fraction of the ball volume occupied by the ionized hot air that form the streamers is ξ). As the rest of the ball is at ambient temperature, the radiated power is proportional to ξ. In the average case considered below ξ turns out to be of the order of 10^{-6}, i.e., about 1 ppm.

VII. EVOLUTION AND DEATH OF THE FIREBALL

As will be seen below, once the fireball is formed in an extremely short time, it begins a slow expansion (which can be qualified as *almost quiescent*) if the helicity is nonvanishing, i.e. if there is linking of magnetic lines and streamers. Let us explain why.

During the almost quiescent expansion, the system appears as a fireball. Note that, even if the streamers are inside a certain sphere, the magnetic field extends farther, although going to zero at infinity. Such an open system cannot be in equilibrium (contrary to a plasma inside a container), so that an expansion starts since its magnetic pressure cannot be completely compensated for. The balance of energy imposes the equality of (a) the energy that the ball loses by expanding, and (b) the energy that it radiates away and that produces its brightness. The magnetic plus kinetic energy can be expressed, for dimensional reasons, as

$$E=\frac{b^2 g_n}{\mu_0 L}, \tag{18}$$

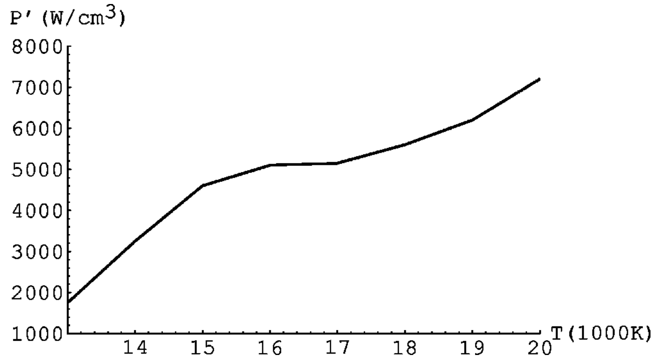

FIG. 2. Power density vs temperature $P'(T)$ emitted by a plasma torch (after Ref. [24]).

g_n being a dimensionless quantity depending on the functional form of $\mathbf{B}(\mathbf{r})$ and the linking number n of the magnetic knot, and L the radius of the ball as defined at the end of Sec. IV. This expansion can be considered part of the relaxation process, since, as the system is open, the minimum energy compatible with the helicity conservation is zero (corresponding to $L=\infty$).

The temperature of the plasma in the streamers is assumed to be in the interval 15 500–18 000 K, where there is a shoulder in the experimental curve $P'(T)$ of the power density radiated by the plasma versus the temperature [24] (see Fig. 2). This explains why the fireballs retain their constant brilliance: if the emission is due to a plasma inside the ball in this range of temperature, it can radiate for some time without appreciably decreasing its brilliance, as far as it is in the shoulder. This is precisely what happens with fireballs: although something is surely cooling inside them, witnesses did not report a decrease of their brightness. As the expansion can be assumed to be adiabatic, the radius L is proportional to $1/\sqrt{T}$, this being the reason for the slowness of the expansion as far as the streamer temperature is in the shoulder. Note, moreover, that this is a plausible range for the temperature, since it is known that the peak temperature in the leader step of an ordinary lightning is in the range 25 000–30 000 K [3]. However, the streamers cool in this expansion, the consequent decrease of the conductivity producing a helicity loss that eliminates the constraint imposed by the conservation of h (see Sec. VIII). As a consequence, the structure is eventually destroyed, and the fireball ends its life.

Let a force-free magnetic knot coupled to the plasma in a ball be formed at $t=0$. Its energy $E=\int B^2/2\mu_0\, d^3r$ (where the kinetic energy of the plasma has been neglected because of the small volume of the streamers) has the form of Eq. (18). It can be written as $E=B_0^2L_0^3/\mu_0\chi$, where $\chi=L(t)/L_0$ is the radius divided by its initial value. This expression serves as a definition of B_0, which we call ‘‘the effective magnetic field.’’ Note that B_0^2 is larger than B_{av}^2, the average value of B^2 at a time $t=0$. In fact, B_0^2 would be equal to the average value of B^2 at a time $t=0$ of a distribution of magnetic energy that would be confined in a sphere of radius L_0, and would have the same total energy. Indeed, as the magnetic field extends necessarily farther than the ball radius L (as explained above), the typical value of B inside the ball is approximately of the order of $3B_0^2L_0^3/2\pi L_B^3$, L_B being the effective radius of the distribution of magnetic energy at $t=0$. If $L_B=2L_0$, then $B_{av}\sim B_0/4$; if $L=1.5B_0$, then $B_{av}\sim B_0/2.6$. This is important: the typical value of the magnetic field inside the ball is smaller than B_0 and, more importantly, the same can be said of the magnetic field where the gradient of magnetic pressure is larger, which certainly occurs outside the border of the visible ball.

The ball therefore expands to decrease its energy. We assume that the expansion is adiabatic; as the air inside the streamers is a monoatomic gas at the temperature that we consider, its adiabatic parameter is $\gamma=5/3$, the temperature then varying as $T=T_0\chi^{-2}$:

$$E=\frac{B_0^2L_0^3}{\mu_0}\left(\frac{T}{T_0}\right)^{1/2}. \tag{19}$$

If ξ is the fraction of the ball volume that is hot (i.e., the volume of the streamers divided by the volume of the ball), the system loses energy according to

$$\frac{dE}{dt}=-\xi P'(T)V, \tag{20}$$

where $V=4\pi L_0^3\chi^3/3$ is the ball volume, from which it follows[1] that

$$-qB_0^2\frac{TdT}{P'(T)}=dt, \tag{21}$$

with $q=3/(8\pi\xi\mu_0T_0^2)$.

Consequently, as the ball expands, its radius $L=\chi L_0$ increases, the energy decreases, and the temperature evolves in time according to the law

$$-qB_0^2\int_{T_0}^{T}\frac{TdT}{P'(T)}=t. \tag{22}$$

As will be seen in Sec. IX, this equation predicts an slow expansion with a lifetime on the order of seconds for average magnetic fields of the order of 0.5 T. We must emphasize that Eq. (22) is valid for all balls with the same values of B_0, ξ, and T_0, independently of the particular expression of the magnetic field $\mathbf{B}(\mathbf{r})$. For this reason, all the numerical results obtained in Ref. [9] for a particular example are valid in the general case shown here. It must be stressed that the force-free configuration is the natural relaxed state, so that Eq. (22) applies to any linked ball (although the phenomenon was illustrated for simplicity in Ref. [9] through an example that is not a force-free field). The lifetime can be defined as the time during which a ball remains in the shoulder of $P'(T)$ (since it cools down quickly afterward). Assuming that the ball begins at the higher border of this shoulder, the energy

[1]In Ref. [9], where this calculation was first given, there are regrettably two misprints: the factor ξ is explained in the text but is lacking in the expressions for dE/dt and q (noted there as γ), and the exponents in the expression for V appear as 2 instead of 3. However, the computation does make use of the right expressions, and is correct.

density only depends on B_0, and the lifetime on B_0 and ξ. They do not depend on other characteristics or on the functional form of the magnetic knot.

VIII. REASONS FOR THE ALMOST QUIESCENT EXPANSION OF THE FIREBALL

An open system of a plasma and a magnetic field cannot be in equilibrium, this being the main difficulty to construct an electromagnetic model of ball lightning. However, in the topological model, the balls are not in stationary equilibrium but in slow expansion, termed also as almost quiescent expansion (hardly appreciable by the excited witnesses). In this section, we consider three reasons for the slowness of the expansion: the formation of a force-free configuration for the magnetic field, the Alfven conditions, and the conservation of the helicity integral. We stress again that it suffices that these three stabilizing effects hold approximately.

The formation of the force-free configuration after an almost instantaneous Taylor relaxation (as discussed in Sec. V) is important because the Lorentz force vanishes in such a state and *the streamers cannot be cut by the pinch effect*. In a different configuration, it would be impossible to have streamers that last for several seconds. Note that the force-free configuration is not an *ad hoc* hypothesis, but corresponds to states with minimum energy, and appears naturally in relaxation processes in astrophysics and tokamaks.

To assess the importance of the Alfven conditions [Eqs. (10)], we must emphasize that the magnetic ball (the region where B is appreciably different from zero) is larger than the visible ball (the smallest sphere that contains the luminous streamers and has radius L). It follows from Eq. (9) that, in the MHD approximation, the streamers are stationary if the Alfven conditions hold along them (even if these conditions are not verified or are meaningless outside the streamers). Of course, they cannot really be stationary for two reasons: the balls can lower their energy by radiation and expansion, and the resistivity, although small, is not zero. However, it is clear that the Alfven conditions provide a stabilizing effect. Note the following: (i) The charges spiral around a magnetic field; in our case they move parallel to it (as noted in Sec. IV), which is a particular case of spiral motion. (ii) In the force-free configuration reached after the relaxation, the magnetic field, the fluid velocity, and the current are parallel. and (iii) The magnetic field is ''attached'' to the streamers by the equation $\mathbf{j}=\boldsymbol{\nabla}\times\mathbf{B}/\mu_0$, so that if the streamers are stabilized, the same thing happens with $\mathbf{B}$, even if the region with higher magnetic pressure is outside the streamers.

Let us consider now the effect of conservation of the helicity integral. Two questions must be well understood: (i) the reason why the helicity is approximately conserved, and (ii) why this has a stabilizing effect.

(i) The time derivative of the helicity is given by Eq. (3). The product $\eta\mathbf{j}$ is zero outside the streamers since no current flows there. It is small inside them, since the conductivity is high at the temperature interval that we consider. Moreover, the volume of the streamers is very small (as will be seen, of the order of about 1 ppm of the total volume of the average ball). However, these facts by themselves do not guarantee that the helicity is conserved long enough. In order to understand what happens, we must consider Eq. (9). If Alfven conditions hold, it is a diffusion equation of the type $\partial u/\partial t=\kappa\nabla^2 u$. As $\mathbf{j}=\nabla\times\mathbf{B}/\mu_0$, the current would diffuse with the magnetic field in such a way that the streamers would widen and the structure be destroyed. The conductivity inside the streamers at the temperature range that we are considering is of the order of $\sigma\approx 10^4$ $\mathrm{Ohm}^{-1}\cdot\mathrm{m}^{-1}$, so that $\kappa=1/\sigma\mu_0\approx 80$ m^2/s. With this value, the diffusion would be too rapid: a simple calculation shows that the streamers would widen and be destroyed too quickly for the model to be correct. This is the same conclusion reached after a naive application of the virial theorem.

However, the previous argument misses an important and essential point: there is a conflict of two diffusions. In order for the current $\mathbf{j}$ to diffuse and widen the streamers, the air between them, which is initially at ambient temperature, must be heated several thousands of kelvin (as current cannot flow in cold air). In other words, the diffusion of the magnetic field and the current cannot take place until the thermal diffusion paves the way. As it is clear that the thermal diffusion is much slower than the electromagnetic one, there is a conflict between the two diffusive processes, in such a way that the time necessary for the heating of the air delays the process of helicity loss and increase the system lifetime by a factor of several orders of magnitude.

In conclusion, the assumption that the helicity is approximately conserved is justified.

(ii) As emphasized above, the conservation of the helicity poses a constraint on the expansion velocity. This is because it closes many decay channels for the balls, this being the reason for its stabilizing effect. The expansion of the ball with $L=L(t)$ in Eq. (4) is clearly allowed by the helicity conservation, as noted at the end of Sec. III, even if $h\neq 0$. On the other hand, this conservation blocks other relaxation channels for which h is not conserved, making more difficult the dissipation of the ball. Let us be precise. Consider the more general class of decays, which would be in principle possible, such as

$$\mathbf{B}=\frac{bL_0^k}{L^{k+2}}\mathbf{f}\left(\frac{\mathbf{r}}{L}\right), \tag{23}$$

with $L=L(t)$ increasing in time, which correspond to the same initial magnetic field. The variation in time of the helicity under expansion (23) is

$$h(t)=\frac{h(0)}{L^{2k}(t)}. \tag{24}$$

As we see, the helicity is only conserved if $k=0$. We must now compare the two cases of (a) a linked ball, $h\neq 0$; and (b) and unlinked ball, $h=0$. If $h\neq 0$, all but one of these expansions are blocked; the only case allowed by the conservation of the helicity is $k=0$, which is the expansion [Eq. (4)] just considered, the evolution being given by [Eq. (22)]. As will be shown in Sec. IX, it is a slow decay.

On the other hand, if $h=0$, all the expansions (23) are then compatible with the conservation of the helicity. None of the channels is blocked.

Note that, repeating the calculations leading to Eq. (22), with Eq. (23) instead of Eq. (4), we obtain

$$-qT_0^{-k}B_0^2(1+2k)\int_{T_0}^{T}\frac{T^{1+k}dT}{P'(T)}=t, \quad (25)$$

that reduces to Eq. (22) if $k=0$. As seen, $t\to 0$ in the limits $k\to -1/2$ and $k\to\infty$, which means that the expansion is instantaneous in those limits. Note that in both cases the system traverses the shoulder in zero time; if $k\to\infty$, the relaxation consisting of the magnetic field goes to zero instantaneously. However, these expansion modes are forbidden by the helicity conservation, if $h\neq 0$.

As seen, there is no ball lightning without linking and helicity, since the system decays too rapidly to be seen. Otherwise stated, linked balls live longer that unlinked balls.

Note that we do not claim that Eq. (23) gives the exact modes of decay, but just particular expansions that show the tendency of the balls to expand much more quickly if there is no linking. It must be remarked, moreover, that the assumption of a spherically symmetric expansion is an approximation of the more complex behavior of real cases, in which the magnetic energy density is not spherically symmetric.

We conclude this section by stating that the virial theorem, which has been used to disprove some electromagnetic models of ball lightning, cannot be applied here because our balls are not in stationary equilibrium. This theorem does not preclude the almost quiescent expansion of our model.

IX. DISCUSSION OF THE MODEL

According to Smirnov [27], the average values of the diameter, power emitted and lifetime of ball lightning are $2L=(28\pm 4)$ cm, $P=(113\pm 16)$ W, and $\tau=10^{0.95\pm 0.25}$ s, respectively. To test the model, we will consider, therefore, the case of a ball of radius $L=15$ cm, emitting a power $P=100$ W, and calculate its lifetime. We assume radiation emission at local thermodynamic equilibrium, and conveniently take the data from argon plasma torch measurements, the most extensively studied case, where the experimental result are best known [24], as described in Fig. 2. Equivalent data in air are known to differ by no more than 10%, which is acceptable at our precission level. A part of the radiation is bremsstrahlung; the rest comes from atomic lines between excited states, from the excited states to the ground state, and from transitions from the continuum. Note the shoulder between about 15 500 and 18 000 K, where the power is almost independent of the temperature. Also that 1 cm^3 of air at this temperature range emits about 5500 W.

Assuming that the streamers inside the ball stay within that temperature range, the power radiated will be almost constant as far as the system remains in the shoulder, even while the streamers temperature decreases. This explains the amazing constancy of the brightness of ball lightnings in our model.

The streamers occupy in this second version of the model a very small part of the ball volume. Assuming a temperature of 18 000 K, as 1 cm^3 of air emits 5500 W, if the power is 100 W, the volume of the streamers must be 1/55 cm^3: just a proportion of about $\xi=1.2\times 10^{-6}$ of the ball volume is ionized and hot. Assuming that the streamers diameter is in the range 50–200 μm, their total length is between about 60 and 900 cm, approximately. In general, it is to be expected that

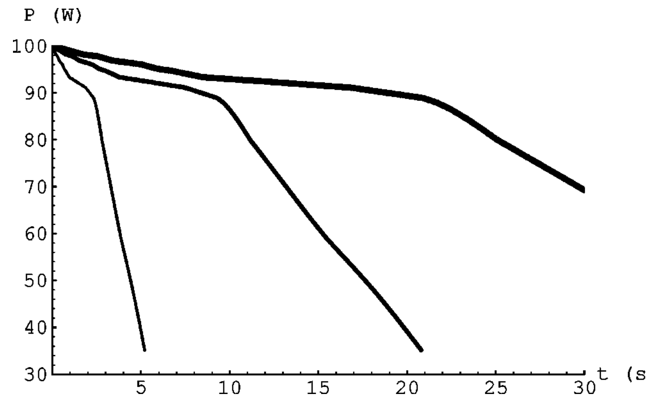

FIG. 3. Shape of the curve $P(t)$, power radiated by the ball vs time, for three values of the magnetic field: $B_0=1$ T (thin line), $B_0=2$ T (medium line), and $B_0=3$ T (thick line) (note that the average magnetic field is smaller than B_0 by a factor on the order of 0.3). The lifetimes are approximately 2.5, 10, and 22 s. The expansion of the ball during its lifetime is very slow, and amounts to just 6% of the radius, so that it is difficult for the witnesses to become aware it.

the system will have angular momentum; this means that a shining line that long, consisting of several linked loops, would be in rotation, this explaining why it is perceived as a fuzzy patch of light.

The evolution of the temperature and, consequently, of $P(t)$, the power radiated by the ball vs time, is easily obtained by integrating Eq. (22) with $\xi=1.2\times 10^{-6}$. The result is plotted in Fig. 3 for $T_0=18\,000$ K and three values of the magnetic field B_0. As can be seen, curve $P(t)$ has the shape that one must expect for ball lightning: the brilliance varies little for a while, and decreases more rapidly afterward. We have defined the lifetime of the ball as the time it takes to traverse the shoulder of the function $P'(T)$, which corresponds to a decrease of about 10% in the radiated power (i.e., the time to go from 100 to 90 W). With this criterion, the lifetime turns out to be $\tau=2.5B_0^2$ (with B_0 in T). As is known, the magnetic field can reach several T near the discharge of a lightning. If $B_0=1.9$ T, the lifetime in this model for radius equal 15 cm is 9 s, equal to the observed average value. Note that, as explained in Sec. VII, this value of the effective field B_0 correponds to a lower value for the field inside the ball, approximately in the interval 0.5–0.7 T.

The value of $\chi=L(t)/L_0$ changes little during the ball lifetime, from 1 to 1.06; this means that the diameter passes from 30 to about 32 cm, a change hardly noticeable since the ball rim is slightly diffuse, not a clearcut line; moreover, the witnesses were excited and impressed. This is thus in agreement with witness reports, while at the same time the balls are in expansion, as they must be in an electromagnetic model.

The average energy of the ball is about 20 kJ, according to Smirnov [27]. In this model, the initial energy of the average case is $E=2.685B_0^2$ kJ. For $B_0=2$ T, this is about 11 kJ; for $B_0=3$ T, it is near 24 kJ; the agreement is thus good (these two values of B_0 correspond to average values of B in the interval 0.7–1.1 T, approximately). Only a part of this energy will be radiated during the time in which the ball shines.

Note that, when the resistivity enters into play, it produces a helicity dissipation according to Eq. (3); moreover, the MHD approximation becomes worse, the last term in Eq. (9) that produces a diffusion of the magnetic field increasing its effect; this accelerates the end of the structure, making the decrease of the power steeper and more abrupt than what is shown in Fig. 3, thus improving the agreement with what was observed by the witnesses. We must emphasize that these calculations depend on an analytical expression of the magnetic field only through the characteristic field B_0.

X. SUMMARY AND CONCLUSIONS

To summarize, the stabilizing effects of (i) the force-free field configuration after a Taylor relaxation process, (ii) the Alfven conditions in the MHD approximation, and (iii) the approximate conservation of the helicity integral (or equivalently, of the linking the magnetic lines and streamers), can be used to construct a realistic model of ball lightning that improves and generalizes the one presented in Refs. [8,9], in which the following hold true,

(1) The fireball of ball lightning is formed near the discharge of an ordinary lightning, if some streamers form closed and linked loops, like the tubes shown in Fig. 1.

(2) During an almost instantaneous process of Taylor relaxation, a state is formed at a time zero consisting in a force-free magnetic knot coupled to the plasma inside the streamers. Because of the force-free condition $(\nabla\times\mathbf{B})\times\mathbf{B}=\mathbf{0}$, the magnetic field is parallel to the current, in such a way that ions and electrons can move along the same streamers (as explained at the end of Sec. IV). Note that the streamers and the magnetic lines have the same linking number. Only a very small part of the ball volume is hot (the plasma in the streamers), the rest being at ambient temperature. In the case studied, there is about a part per million of plasma.

(3) After the formation of the force-free configuration, the relaxation process goes on, the system radiating away energy and expanding slowly its radius in a process called here *almost quiescent expansion*, while verifying the Alfven conditions. The system is seen then as a fireball. The high stability of the balls is explained as a consequence of the Alfven conditions and of the constraint imposed by the helicity conservation (in other words, by the linking of the magnetic lines and the streamers). If the system is linked (i.e., if the helicity is nonzero), the expansion turns out to be so slow that it could not be appreciated by the witnesses. This is because many rapid expansion channels are closed, as they violate the helicity conservation. But these channels are open if the system is unlinked, a case in which the system is not seen, as it decays almost instantaneously. The end of the fireball is due to the cooling of the plasma, which starts a process of progressive increase of the resistivity and of helicity loss. Note that, since the Lorentz force vanishes, there can be no pinch effect on the streamers. This adds stability to the system.

(4) The temperature of the plasma in the streamers is in the interval 15 500–18 000 K, where there is a shoulder in the curve $P'(T)$ of the power density radiated by the plasma versus the temperature. This explains why the fireballs keep constant their brilliance: when the plasma in the streamers cools, it goes to the left along the shoulder without changing its radiance appreciably while the temperature remains in this interval. Furthermore, if the expansion is adiabatic, the radius of the ball is proportional to $1/\sqrt{T}$, so that it changes little during the expansion.

(5) In this model the fireball's lifetime is much longer than the resistive time. This is because the tendency of the current along the streamers to diffuse, with the consequent destruction of the structure, is counteracted by the much slower velocity of the thermal diffusion. The streamers cannot widen before the intermediate air is heated several thousands of kelvin and this takes time. This conflict between the two diffusive processes provides an essential stability factor that lengthens the lifetime by several orders of magnitude. The usual arguments against the electromagnetic models of ball lightning, which are based on the virial theorem, do not consider this effect and cannot be applied to this model.

(6) The model is in good agreement with the observed lifetime, energy and radiated power of the fireballs. The streamers occupy a fraction of the ball volume of the order of $\xi=10^{-6}$, corresponding to several meters of shining line. As this line consists of tangled streamers and, in the general case, it rotates because of its angular momentum, it must be seen as a diffuse and continuous patch of light.

This model also explains two meaningful and significant observations. First, in some cases filaments are observed trailing a ball; they must be streamers which break open and follow behind (see the photographs in p. 10 of Ref. [4] and in Chap. 5 of Ref. [2]). Second, as stated above, some witnesses claimed that ball lightning is cold, while other witnesses were burned. To explain this, the important point is that the power radiated by the fireballs is just of the order of 10–150 W in this model, in spite of the plasma being hot, because only a small fraction ξ of the ball volume is ionized. Note that it is impossible that the entire ball consists of hot plasma, since the output would be enormous, on the order of 10–100 MW. The fact that only a small fraction ξ of the ball is hot thus solves the problem of the order of magnitude of the output. This contradictions among the witnesses are thus solved by this model. Because the output is on the order of 100 W and only a small part of the ball is hot, the balls can burn a person or start a fire if there is contact, but no feeling of heat is produced if there is not.

An important and difficult problem is the production of fireballs in the laboratory. This has been attempted by several means, combustion of mixtures of gases for instance; the best results in air were the fireballs produced by Ohtsuki and Ofuruton [28] in 1991 by the interference of microwaves. They are similar to ball lightning but it is not certain that they are the same thing. This model suggests a way of producing fireballs: with two discharges orthogonal or at least transverse to one another, and strong enough according to the data of Ref. [11]. The combination of magnetic fields around the discharges should make the formation of linked lines easier. The probability could be enhanced by rotating the electrodes very rapidly.

ACKNOWLEDGMENTS

We are grateful to M. V. Berry, F. Blanco, D. Bouwmeester, D. K. Callebaut, A. Ibort, J. M. Montesinos, S. Singer, A. Tiemblo, and T. Tuomi for suggestions and encouragement.

[1] S. Singer, *The Nature of Ball Lightning* (Plenum Press, New York, 1971).
[2] J.D. Barry, *Ball and Bead Lightning* (Plenum Press, New York, 1980).
[3] M.A. Uman, *Lightning* (Dover, New York, 1984).
[4] *The Science of Ball Lightning*, edited by Y.H. Ohtsuki (World Scientific, Singapore, 1989).
[5] M. Stenhoff, *Ball Lightning. An Unsolved Problem in Atmospheric Physics* (Kluwer, Dordrecht, 1999).
[6] *Proceedings of the 6th International Symposium on Ball Lightning (ISBL99)*, edited by G. C. Dijkhuis (University of Antwerp, Antwerp, 1999).
[7] *Balls of Fire, Recent Research in Ball Lightning*, edited by S. Singer (Springer Verlag, Berlin, in press).
[8] A.F. Rañada and J.L. Trueba, Nature (London) **383**, 32 (1996).
[9] A.F. Rañada, M. Soler, and J.L. Trueba, J. Geophys. Res. **103**, 23 309 (1998).
[10] K. Lust and A. Schlute, Z. Astrophys. **34**, 263 (1954).
[11] I. Alexeff and M. Rader, Fusion Technol. **27**, 271 (1995).
[12] A.F. Rañada, J. Phys. A **23**, L815 (1990).
[13] A.F. Rañada, Lett. Math. Phys. **18**, 97 (1989); J. Phys. A **25**, 1621 (1992).
[14] A.F. Rañada and J.L. Trueba, Phys. Lett. **202A**, 337 (1995); **232A**, 25 (1997).
[15] H.K. Moffatt, J. Fluid Mech. **35**, 117 (1969).
[16] A.F. Rañada, Eur. J. Phys. **13**, 70 (1992).
[17] C.L. Rousculp and R.L. Stenzel, Phys. Rev. Lett. **79**, 837 (1997).
[18] R.L. Ricca, D.C. Samuels, and C.F. Barenghi, J. Fluid Mech. **391**, 29 (1999).
[19] G. Marsh, *Force Free Magnetic Fields: Solutions, Topology and Applications* (World Scientific, Singapore, 1996).
[20] S. Chandrasekhar and L. Woltjer, Proc. Natl. Acad. Sci. U.S.A. **44**, 285 (1958); L. Woltjer, *ibid.* **44**, 489 (1958).
[21] D. Voslamber and D.K. Callebaut, Phys. Rev. **128**, 2016 (1962).
[22] D.K. Callebaut (private communication).
[23] J.B. Taylor, Rev. Mod. Phys. **58**, 741 (1986).
[24] D.L. Evans and R.S. Tankin, Phys. Fluids **10**, 1137 (1967).
[25] H. Raether, Z. Phys. **112**, 464 (1939).
[26] I. Gallimberti, Pure Appl. Chem. **60**, 663 (1988).
[27] B.M. Smirnov, Phys. Rep. **188**, 1 (1990).
[28] Y.H. Ohtsuki and H. Ofuruton, Nature (London) **350**, 139 (1991).